INVERSE TRIGONOMETRIC FUNCTION

EXERCISE – I HINTS & SOLUTIONS

Sol.1 D

$$f(x) = \underbrace{\cos^{-1}x}_{[-1,1]} + \underbrace{\cot^{-1}x}_{R} + \underbrace{\text{cosec}^{-1}x}_{|x| \ge 1}$$

$\cap$

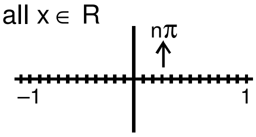
$x \in \{-1, 1\}$

Sol.2 D

$f(x) = \text{cosec}^{-1}(\cos x)$ for all $x \in R$

$\downarrow$

-1 to 1



since curve of $\text{cosec}^{-1} x$ takes value for $x \le -1$ or $x \ge 1$

so $x \in n\pi \rightarrow$ (multiple of π)

Sol.3 C

$$f(x) = \underbrace{\sin^{-1}x}_{[-1,\ 1]} + \underbrace{\tan^{-1}x}_{R} + \underbrace{\sec^{-1}x}_{|x| \ge 1}$$

$D_f : x \in \{-1, 1\}$

so $R_f = \{f(-1), f(1)\}$

$$f(-1) = \frac{-\pi}{2} \ \frac{-\pi}{4} + \pi \quad = \frac{\pi}{4}$$

$$f(1) = \frac{\pi}{2} + \frac{\pi}{4} \quad = \frac{3\pi}{4}$$

$$\text{so} \quad R_f : \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$$

Sol.4 D

$$x \ge 0, \qquad \theta = \underbrace{\sin^{-1}x}_{[-1,1]} + \underbrace{\cos^{-1}x}_{[-1,1]} - \underbrace{\tan^{-1}x}_{x \in R}$$

but $x \ge 0$ so, $x \in [0, 1]$

$$\theta = \frac{\pi}{2} - \tan^{-1}x$$

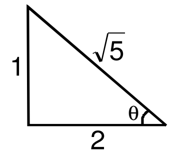
$$R_\theta : \left[\theta\big|_{x=1}, \theta\big|_{x=0}\right] = \frac{\pi}{4} \le x \le \frac{\pi}{2}$$

Sol.5 B

$$y = \cos\left[\tan^{-1}\{\sin(\cot^{-1}\sqrt{3})\}\right]$$

$$y = \cos\left[\tan^{-1}\left\{\sin\left(\frac{\pi}{6}\right)\right\}\right] = \cos\left\{\tan^{-1}\left(\frac{1}{2}\right)\right\}$$

let $\tan^{-1}\dfrac{1}{2} = \theta \Rightarrow \tan\theta = \dfrac{1}{2}$

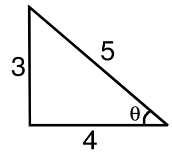


$$\cos\theta = \frac{2}{\sqrt{5}}$$

Sol.6 D

$$y = \tan\left[\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$$

Let $\theta = \sin^{-1}\dfrac{3}{5} \Rightarrow \sin\theta = \dfrac{3}{5}$



$$\tan\theta = \frac{3}{4}$$

$$\text{so} \quad \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\text{so} \quad y = \tan\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$$

$$y = \frac{\tan\left\{\tan^{-1}\left(\frac{3}{4}\right)\right\} + \tan\left\{\tan^{-1}\left(\frac{2}{3}\right)\right\}}{1 - \tan\left(\tan^{-1}\left(\frac{3}{4}\right)\right).\tan\left(\tan^{-1}\left(\frac{2}{3}\right)\right)}$$

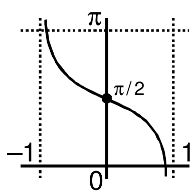
$$y = \frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4}.\frac{2}{3}} = \frac{9+8}{6} = \frac{17}{6}$$

Sol.7 A

$$\sum_{i=1}^{n} \cos^{-1}\alpha_i = 0$$

$\cos^{-1}\alpha_1 + \cos^{-1}\alpha_2 + \ldots\ldots. + \cos^{-1}\alpha_n$

so, $\cos^{-1}\alpha_1$ is always true



so $\cos^{-1}\alpha_1 = \cos^{-1}\alpha_2 = \ldots.. = \cos^{-1}\alpha_n = 0$

$$\Rightarrow \quad \cos^{-1}\alpha_1 = 0 \quad \Rightarrow \quad \alpha_1 = 1$$
$$\cos^{-1}\alpha_2 = 0 \quad \Rightarrow \quad \alpha_2 = 1$$
$$\vdots \qquad\qquad \vdots$$
$$\cos^{-1}\alpha_n = 0 \quad \Rightarrow \quad \alpha_n = 1$$

$$\therefore \sum_{i=1}^{m} \alpha_i = n$$

Sol.8 C

$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4}-\frac{\cos^{-1}x}{2}\right)$$

Let $\cos^{-1} x = \theta \quad \Rightarrow \quad x = \cos\theta$

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$\Rightarrow \frac{\left(1+\tan\frac{\theta}{2}\right)^2+\left(1-\tan\frac{\theta}{2}\right)^2}{1-\tan^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{2\left(\sec^2\frac{\theta}{2}\right)}{\left(1-\tan^2\frac{\theta}{2}\right)} = \frac{2}{\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{2}{x}$$

Sol.9 B

$$\cot^{-1}\left(\frac{n}{\pi}\right) > \frac{\pi}{6}, \ n \in N$$

$$\frac{n}{\pi} < \cot\frac{\pi}{6} \ \Rightarrow \ n < \sqrt{3}\,\pi$$

$$\Rightarrow \ n < \sqrt{3} \times 3.14 \quad \Rightarrow \quad n = 5$$

Sol.10 D

$x^2 + ax + \sin^{-1}(x^2 - 4x + 5) + \cos^{-1}(x^2 - 4x + 5) = 0$
consider $(x^2 - 4x + 5)$ let it be equal to $h(x)$, i.e.,
$h(x) = x^2 - 4x + 5$
$D = 16 - 20 < 0$

Min value $= -\dfrac{D}{4a} = \dfrac{4}{4.1} = 1$

Means $(x^2 - 4x + 5) \ge 1$
but $\sin^{-1}x$ & $\cos^{-1}x$ are N.D. for $x > 1$
so, the whole equation is satisfied for $x = 2$ only.

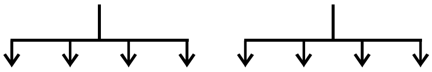
$$4 + 2a + \frac{\pi}{2} = 0$$

$$2a = -4 - \frac{\pi}{2}$$

$$a = -2 - \frac{\pi}{4}$$

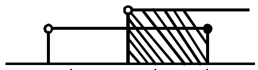
Sol.11 C

$$\underset{(0,\ \pi)}{[\cot^{-1} x]} + \underset{[0,\ \pi]}{[\cos^{-1} x]} = 0$$



0 1 2 3 0 1 2 3

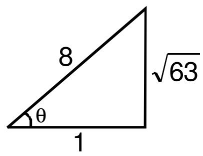
so, $0 < \cot^{-1} x < 1$ & $0 \le \cos^{-1} x < 1$
$\cot 1 < x < \infty$ & $\cos 1 < x \le 1$



so, $x \in (\cot 1, 1]$

Sol.12 A

$$y = \cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right)$$



Let, $\cos^{-1}\frac{1}{8} = \theta \Rightarrow \cos\theta = \frac{1}{8}$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\frac{1}{8}}{2}} \quad \Rightarrow \quad \sqrt{\frac{9}{16}} = \frac{3}{4}$$

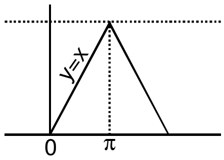
$$\& \quad \cos\frac{\theta}{2} = \cos\left(\frac{\cos^{-1}\left(\frac{1}{8}\right)}{2}\right) - \frac{3}{4}$$

Sol.13 D

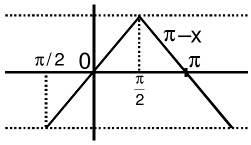
$y = \sin^{-1}[\cos\{\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\}]$

given $\quad x \in \left(\frac{\pi}{2}, \pi\right) \ \Rightarrow \ \frac{\pi}{2} < x < \pi$

Now $\quad \cos^{-1}(\cos x) = x$



$\sin^{-1}(\sin x) = \pi - x$



so, $y = \sin^{-1}[\cos\{x + \pi - x\}]$

$y = \sin^{-1}(\cos \pi) \Rightarrow \sin^{-1}(-1) \Rightarrow -\dfrac{\pi}{2}$

Sol.14 B

Given, $\sin^{-1} x + \sin^{-1} y = \dfrac{2\pi}{3}$,

$\dfrac{\pi}{2} - \cos^{-1} x + \dfrac{\pi}{2} - \cos^{-1} y = \dfrac{2\pi}{3}$

$\cos^{-1} x + \cos^{-1} y = \pi - \dfrac{2\pi}{3} = \dfrac{\pi}{3}$

Sol.15 B

If $x < 0$, $\tan^{-1} x + \tan^{-1} \dfrac{1}{x}$

$y = \tan^{-1}\left(\dfrac{x + \frac{1}{x}}{1 - x.\frac{1}{x}}\right) \Rightarrow \tan^{-1}\left(\dfrac{2x+1}{0}\right)$

If x is less than zero then

$y = -\tan^{-1}(\infty) \Rightarrow -\dfrac{\pi}{2}$

Sol.16 C

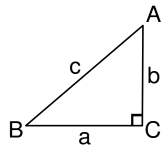
$\tan^{-1} a + \tan^{-1} b,\ a > 0,\ b > 0,\ ab > 1$

$= \pi + \tan^{-1}\left(\dfrac{a+b}{1-ab}\right)$

Sol.17 A

$\tan^{-1}\left(\dfrac{a}{b+c}\right) + \tan^{-1}\left(\dfrac{b}{c+a}\right)$ if $\angle C = 90º$

$\tan^{-1}\left\{\dfrac{\frac{a}{b+c} + \frac{b}{c+a}}{1 - \frac{a.b}{(b+c)(c+a)}}\right\}$



$a^2 + b^2 = c^2$

$\tan^{-1}\left\{\dfrac{ac + a^2 + b^2 + bc}{bc + ab + c^2 - ab + ac}\right\}$

$\Rightarrow \tan^{-1}\left\{\dfrac{c(a+b+c)}{c(a+b+c)}\right\} \Rightarrow \dfrac{\pi}{4}$

Sol.18 D

$\tan^{-1}\left(\dfrac{\sqrt{x^2+1}-1}{x}\right) = \dfrac{\pi}{45º}$

put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x,\ \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$\tan^{-1}\left(\dfrac{|\sec\theta| - 1}{\tan\theta}\right) = \dfrac{\pi}{45º}$

(but $\sec\theta$ is +ve for $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$)

$\tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right) = \dfrac{\pi}{45º}$

$\tan^{-1}\left(\tan\dfrac{\theta}{2}\right) = \dfrac{\pi}{45º}$

Now, $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$\left(-\dfrac{\pi}{4} < \dfrac{\theta}{2} < \dfrac{\pi}{4}\right)$

$\dfrac{\theta}{2} = \dfrac{\pi}{45º} \Rightarrow \tan^{-1} x = 2 \times \dfrac{\pi}{45º} = 8º$

$x = \tan 8º$

Sol.19 B

$y = \cot^{-1}\left(\dfrac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right)$

given $\dfrac{\pi}{2} < x < \pi$

$$y = \cot^{-1}\left(\frac{\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|+\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right|}{\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|-\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right|}\right)$$

$\Rightarrow$ Now if $\frac{\pi}{2} < x < \pi \;\Rightarrow\; \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Now $\sin\frac{x}{2} > \cos\frac{x}{2}$ so, modulus will open directly

$$y = \cot^{-1}\left(-\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right) = \cot^{-1}\left(-\tan\frac{x}{2}\right)$$

$$y = \pi - \cot^{-1}(\tan\frac{x}{2}) \;\Rightarrow\; \pi - \cot^{-1}\cot\left(\frac{\pi}{2}-\frac{x}{2}\right)$$

$$y = \pi - \frac{\pi}{2} + \frac{x}{2} = \frac{\pi}{2} + \frac{x}{2}$$

Sol.20 B

$$y = \tan^{-1}\left(\frac{1-x}{1+x}\right),\; 0 \le x \le 1$$

put $x = \cos\theta \;\Rightarrow\; \theta = \cos^{-1}x \quad \theta \in [0, \pi]$

$$y = \tan^{-1}\left(\frac{1-\cos\theta}{1+\cos\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

here given, $0 \le x \le 1$

$0 \le \cos\theta \le 1$

$0 < \theta \le \frac{\pi}{2}$

comes in PVR of $\theta = \cos^{-1}x$

$0 < \frac{\theta}{2} \le \frac{\pi}{4}$

so, $y = \frac{\theta}{2} = \frac{\cos^{-1}x}{2}$

Now $y_{min} = \left.\frac{\theta}{2}\right|_{\theta=0} = 0$

$y_{max} = \left.\frac{\theta}{2}\right|_{\theta=\frac{\pi}{2}} = \frac{\pi}{4}$

so, $\left(0, \frac{\pi}{4}\right)$

Sol.21 A

$[\cot^{-1}x]^2 - 6[\cot^{-1}x] + 9 \le 0$

let $[\cot^{-1}x] = t$

$t^2 - 6t + 9 \le 0 \;\Rightarrow\; (t-3)^2 \le 0$

$t - 3 = 0 \;\Rightarrow\; [\cot^{-1}x] = 3$

$3 \le \cot^{-1}x < \pi$

$\cot\pi < x \le \cot 3$

$x \in R\ (-\infty, \cot 3]$

Sol.22 B

$$\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{3\sin 2\theta}{5+4\cos 2\theta} = 1$$

$$\Rightarrow \frac{6\tan\theta}{\tan^2\theta+9} = 1$$

$\Rightarrow \tan^2\theta - 6\tan\theta + 9 = 0$

$\Rightarrow (\tan\theta - 3)^2 = 0$

$\tan\theta = 3$

Sol.23 C

$$\cos^{-1}\left\{\frac{x^2}{2}+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right\} = \cos^{-1}\frac{x}{2} - \cos^{-1}x$$

The above holds iff

$1 \ge x \ge 0 \quad \& \quad 1 \ge \frac{x}{2} \ge 0$

$0 \le x \le 1 \quad \& \quad 0 \le x \le 2$

$\cap$

$0 \le x \le 1$

Sol.24 C

$$\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$$

$$\cos^{-1}(\sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p}) + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$$

$$\cos^{-1}0 + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$$

$$\cos^{-1}\sqrt{1-q} = \frac{\pi}{4} \;\Rightarrow\; 1 - q = \frac{1}{2}$$

$\Rightarrow \quad q = \frac{1}{2}$

so, $0 \le \sqrt{p} \le 1$ & $0 \le \sqrt{1-p} \le 1$

$0 \le p \le 1$ & $0 \le 1 - p \le 1$

$-1 \le -p \le 0 \quad \Rightarrow \quad 0 \le p \le 1$

Sol.25 A

Sol.26 A

$u = \cot^{-1}\sqrt{\tan\alpha} - \tan^{-1}\sqrt{\tan\alpha}$

then, $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = ?$

$u = \frac{\pi}{2} - 2\tan^{-1}(\sqrt{\tan\alpha})$

Now $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{4} + \tan^{-1}\sqrt{\tan\alpha}\right)$

$\Rightarrow \quad \sqrt{\tan\alpha}$

Sol.27 B

$\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2\cos^{-1}x = \frac{\pi}{2} - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$2\cos^{-1}x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

let $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \cos\theta = \frac{1}{2}$

so, $\cos^{-1}x = \frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\theta}{2}$

$x = \cos\left(\frac{\theta}{2}\right) \Rightarrow x = \frac{1+\cos\theta}{2}$

$\Rightarrow \quad x = \frac{3}{4}$ hence unique solution

Sol.28 C

$\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$

$\frac{\pi}{3} - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = 0$

$\sqrt{\frac{3}{x}} = \sqrt{\frac{3}{2}} \Rightarrow \frac{3}{x} = \frac{3}{4}$

$\Rightarrow \quad x = 4$

Sol.29 D

$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Let $x = \sin\theta$

$2\sin^{-1}(\sin\theta) = \sin^{-1}(\sin 2\theta)$

$2\theta = \sin^{-1}(\sin 2\theta)$, possible if

$-\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$-\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$

$-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

Sol.30 B

$\sin^{-1}x + \cos^{-1}(1-x) = \sin^{-1}(-x)$

$2\sin^{-1}x = -\cos^{-1}(1-x)$

$\cos(2\sin^{-1}x) = \cos(\cos^{-1}(1-x))$

$1 - 2x^2 = 1 - x$

$2x^2 - x = 0 \Rightarrow x(2x-1) = 0$

$\Rightarrow x = 0, \frac{1}{2}$

check for $x = \frac{1}{2} \to$ rejected

$x = 0 \to$ accept (1 solution)

Sol.31 B

$\sin^{-1}\left(\frac{x}{5}\right) + \mathrm{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\sin^{-1}\left\{\frac{x}{5}\times\frac{3}{5}+\frac{4}{5}\times\frac{\sqrt{25-x^2}}{5}\right\} = \frac{\pi}{2}$

$\frac{3x}{25} + \frac{4\sqrt{25-x^2}}{25} = 1$

$x^2 - 6x + 9 = 0 \Rightarrow (x-3)^2 = 0 \Rightarrow x = 3$

Sol.32 A

$\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$

$(\sin^{-1} x + \sin^{-1}(1-x)) = \cos^{-1} x$

$\sin^{-1}(x\sqrt{1-(1-x)^2} + (1-x)\sqrt{1-x^2}) = \sin^{-1}\sqrt{1-x^2}$

$x\sqrt{2x-x^2} = \sqrt{1-x^2}\ (1-1+x)$

$x^2(2x-x^2) - (1-x^2)x^2 = 0$

$2x^3 - x^2 = 0$

$x^2(2x-1) = 0 \Rightarrow x = 0, \frac{1}{2}$

Both accepted.

Sol.33 C

$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$\tan^{-1}\left\{\frac{1+x+1-x}{1-(1-x^2)}\right\} = \frac{\pi}{2}$

$\frac{2}{2x-x^2} = \infty \qquad \Rightarrow 2x - x^2 = 0$

$\Rightarrow x = 0$

Sol.34 B

$\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

$-2\sin^{-1} x = \cos^{-1}(1-x)$

$1 - 2x^2 = 1 - x$

$2x^2 - x = 0 \qquad \Rightarrow \qquad x = 0, \frac{1}{2}$

check : $x = 0$

L.H.S. $\Rightarrow \frac{\pi}{2} - 0 \qquad = \frac{\pi}{2} = \text{RHS}$

$x = \frac{1}{2}$

$\frac{\pi}{6} - \frac{\pi}{3} \Rightarrow -\frac{\pi}{2} \Rightarrow$ so rejected

Sol.35 B

$\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\tan^{-1}\left\{\frac{\frac{1}{2x+1}+\frac{1}{4x+1}}{1-\frac{1}{(2x+1)(4x+1)}}\right\} = \tan^{-1}\frac{2}{x^2}$

$\frac{4x+1+2x+1}{(2x+1)(4x+1)-1} = \frac{2}{x^2}$

$\frac{6x+2}{(8x^2+6x)} = \frac{2}{x^2} \qquad \Rightarrow 6x^3 + 2x^2 = 16x^2 + 12x$

$\Rightarrow 6x^3 - 14x^2 - 12x = 0$

$\Rightarrow x(6x^2 - 14x - 12) = 0$

EXERCISE – II HINTS & SOLUTIONS

Sol.1 A,B

$\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \dfrac{3\pi}{2}$

above relation is possible only when

$\sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \dfrac{\pi}{2}$

$x = y = z = 1$

Sol.2 C,D

$x^2 - x - 2 > 0$
$\alpha^2 - \alpha - x > 0$
$\alpha^2 - 2\alpha + \alpha - 2 > 0$
$\alpha(\alpha - 2) + 1(\alpha - 2) > 0$
$\alpha(\alpha - 2) + 1(\alpha - 2) > 0$
$\alpha \in (-\infty, 1) \cup (2, \infty)$

Sol.3 B,D

$6\sin^{-1}\left(x^2 - 6x + \dfrac{17}{2}\right) = \pi$

$\sin^{-1}\left(x^2 - 6x + \dfrac{17}{2}\right) = \dfrac{\pi}{6}$

$x^2 - 6x + \dfrac{17}{2} - \dfrac{1}{2} = 0$

$x = 2, 4$

Sol.4 A,B,C

$\tan\left(\cos^{-1}\left(\dfrac{4}{5}\right) + \tan^{-1}\left(\dfrac{2}{3}\right)\right) = \dfrac{a}{b}$

$\tan\left(\tan^{-1}\left(\dfrac{3}{4}\right) + \tan^{-1}\left(\dfrac{2}{3}\right)\right) = \dfrac{a}{b}$

$\tan\left(\tan^{-1}\left(\dfrac{17}{6}\right)\right) = \dfrac{a}{b}$

$\dfrac{a}{b} = \dfrac{17}{6} \quad \Rightarrow \quad a - b = 11$

$\Rightarrow \quad a + b = 23$
$\Rightarrow \quad 3b = 3.6 = 18 = a + 11$

Sol.5 B,C,D

$y = \cos\left[\dfrac{1}{2}\cos^{-1}\left\{\cos\left(\dfrac{-14\pi}{5}\right)\right\}\right]$

$y = \cos\left[\dfrac{1}{2}\cos^{-1}\left\{\cos\left(\dfrac{14\pi}{5}\right)\right\}\right]$

$y = \cos\left[\dfrac{1}{2}\cos^{-1}\left\{\cos\left(\dfrac{4\pi}{5} + 2\pi\right)\right\}\right]$

$y = \cos\left[\dfrac{1}{2}\cos^{-1}\left\{\cos\dfrac{4\pi}{5}\right\}\right]$

$y = \cos\left(\dfrac{1}{2} \times \dfrac{4\pi}{5}\right) \quad \Rightarrow \quad \cos\left(\dfrac{2\pi}{5}\right)$

$y = \sin\left(\dfrac{\pi}{2} - \dfrac{2\pi}{5}\right) \quad \Rightarrow \quad \sin\left(\dfrac{\pi}{10}\right)$

$y = \cos\left(\pi - \dfrac{3\pi}{5}\right) \quad \Rightarrow \quad -\cos\left(\dfrac{3\pi}{5}\right)$

Sol.6 A,C

Given $a = \sin^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) + \cos^{-1}\left(\dfrac{-1}{2}\right)$

$b = \tan^{-1}\left(-\sqrt{3}\right) - \cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$

$a = -\dfrac{\pi}{4} + \pi - \dfrac{\pi}{3} \quad \Rightarrow \quad \dfrac{5\pi}{12}$

$b = -\dfrac{\pi}{3} - \pi + \dfrac{\pi}{3} \quad \Rightarrow \quad -\pi$

$a - b = \dfrac{17\pi}{12}$ & $a + b = -\dfrac{7\pi}{12}$

Sol.7 A,C

$f(x) = \sin^{-1} x + \cos^{-1} x$

Now f(x) will be equal to $\dfrac{\pi}{2}$ iff arg lie b/w –1 to 1

Sol.8 A,C,D

$\text{cosec}^{-1} x = \sin^{-1}\dfrac{1}{x}$

$|x| \geq 1$

Sol.9 A,B,C

given $0 < x < 1$ then

$$y = \tan^{-1} \frac{\sqrt{1-x^2}}{(1+x)}$$

put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$\theta \in [0, \pi]$

$$y = \tan^{-1}\left(\frac{|\sin\theta|}{1+\cos\theta}\right) = \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$$

($\because$ $\sin\theta$ is +ve in 1st & 2nd quadrant)

$$y = \tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

Now, given , $0 < x < 1$

$0 < \cos\theta < 1$

$$0 < \theta < \frac{\pi}{2} \Rightarrow \left(0 < \frac{\theta}{2} < \frac{\pi}{4}\right)$$

$$\text{so, } y = \tan^{-1}\tan\left(\frac{\theta}{2}\right) = \frac{\theta}{2} = \left(\frac{\cos^{-1}x}{2}\right)$$

Now in (B) & (C) put $x = \cos\theta$

$$y = \cos^{-1}\left|\cos\frac{\theta}{2}\right| \Rightarrow \cos^{-1}\left(\cos\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$\Rightarrow \left(\cos^{-1}\frac{x}{2}\right)$$

$$y = \sin^{-1}\left|\sin\frac{\theta}{2}\right| \Rightarrow \sin^{-1}\sin\frac{\theta}{2} \Rightarrow \frac{\theta}{2} = \left(\frac{\cos^{-1}x}{2}\right)$$

Sol.10 B,C

Given : $\alpha = 2\tan^{-1}(\sqrt{2} - 1)$

$$\beta = 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\gamma = \cos^{-1}\left(\frac{1}{3}\right)$$

$\alpha = 2 \times 22.5^\circ = 45^\circ$

$$\beta = \frac{3\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{12} = 105^\circ$$

$$\gamma = \cos^{-1}\left(\frac{1}{3}\right) = \cos^{-1}(0.33)$$

$\alpha < \gamma < \beta$

so, $\gamma > \alpha$ $\qquad$ $\beta > \gamma$

Sol.11 B,C

$2x = \tan(2\tan^{-1} a) + 2\tan(\tan^{-1} a + \tan^{-1} a^3)$

$2x = \tan(\tan^{-1} a + \tan^{-1} a) + 2\tan(\tan^{-1} a + \tan^{-1} a^3)$

$$2x = \frac{2a}{1-a^2} + 2\frac{a+a^3}{(1-a^2)(1+a^2)}$$

$$x = \frac{a}{1-a^2} + \frac{a}{(1-a^2)}$$

$a^2x - x + 2a = 0$ $\qquad$ (A) is valid

& $a \neq -1$ & 1 $\qquad$ (D) is valid

Sol.12 A,C

$\cos^{-1} x = \tan^{-1} x$

let $\cos^{-1} x = \theta = x = \cos\theta$

$$\tan\theta = \frac{\sqrt{1-x^2}}{x}$$

so, $\theta = \tan^{-1} x$

$$\tan^{-1}\frac{\sqrt{1-x^2}}{x} = \tan^{-1}x$$

$$\frac{\sqrt{1-x^2}}{x} = x \Rightarrow x^4 + x^2 - 1 = 0$$

$$x^2 = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow x^2 = \frac{\sqrt{5}-1}{2} \text{ or } \frac{-\sqrt{5}-1}{2}$$

Now, $\sin(\cos^{-1} x) = \sin\theta = \tan\theta \times \cos\theta = x^2$

$$\sin(\cos^{-1} x) = \frac{\sqrt{5}-1}{2}$$

Sol.13 A,D

$$\sum_{n=1}^{\infty} \tan^{-1}\frac{4n}{(n^4 - 2n^2 + 2)}$$

$$= \sum_{h=1}^{\infty} \tan^{-1}\left\{\frac{(n+1)^2 - (n-1)^2}{1 + (n+1)^2)(n-1)^2}\right\}$$

$$= \sum_{n=1}^{\infty} \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$$

$$= - \tan^{-1} 1 = -\frac{\pi}{4}$$

EXERCISE – III

HINTS & SOLUTIONS

Sol.1 From the graph :

$f(-3) = 2$, $|f(-1)| = 2$, $\left[f\left(\frac{7}{8}\right)\right] = -2$

$f(0) = 0$, $\cos^{-1}(f(-2)) = 0$

$f(-7) = f(-7 + 8) = f(1) = -2$

$f(20) = f(12 + 8) = f(12) = f(4 + 8) = f(4)$

$\Rightarrow \quad f(20) = f(4) = 3$

Now putting all values

$2 + 4 - 2 + 0 + 0 - 2 + 3 = 5$

Sol.2 (i) $\sin^{-1} x > -1$

$1 \geq x > -\sin 1 \quad \Rightarrow \quad x \in (-\sin 1, 1]$

(ii) $\cos^{-1} x < 2 \quad \Rightarrow \cos x > \cos 2$

but $x \leq 1$, so $x \in (\cos 2, 1]$

(iii) $\cot^{-1} x < -\sqrt{3}$

No solution because curve of $\cot^{-1} x$ never lies below origin.

Sol.3 (i) $y = \sin^{-1}\left(\sin\left(\frac{7\pi}{6}\right)\right)$

$y = \sin^{-1}\left\{\sin\left(\pi + \frac{\pi}{6}\right)\right\}$

$y = \sin^{-1}\left\{\sin\left(-\frac{\pi}{6}\right)\right\} = -\frac{\pi}{6}$

(ii) $y = \tan^{-1}\left(\tan\left(\frac{2\pi}{3}\right)\right) = \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{3}\right)\right)$

$y = -\frac{\pi}{3}$

(iii) $y = \cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

$y = \cos^{-1}\left\{\cos\left(\pi + \frac{\pi}{4}\right)\right\}$

$y = \cos^{-1}\left\{-\cos\frac{\pi}{4}\right\} = \pi - \frac{\pi}{4} \Rightarrow \frac{3\pi}{4}$

(iv) $y = \sec^{-1}\left(\sec\frac{7\pi}{4}\right)$

$y = \sec^{-1}\left\{\sec\left(\pi + \frac{3\pi}{4}\right)\right\}$

$y = \sec^{-1}\left\{-\sec\frac{3\pi}{4}\right\} = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

Sol.4 (i) $y = \sin^{-1}(\sin 5) = 5 - 2\pi$

(ii) $y = \cos^{-1}(\cos 10) = 4\pi - 10$

(iii) $y = \tan^{-1}(\tan(-6)) = -\tan^{-1}(\tan 6) = 2\pi - 6$

(iv) $y = \cot^{-1}(\cot(-10)) \Rightarrow \pi - \cot^{-1}(\cot 10)$

$\Rightarrow \pi - 10 + 3\pi \Rightarrow 4\pi - 10$

(v) $y = \cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right\}$

$= y = \cos^{-1}\left\{\cos\frac{\pi}{4}\cos\frac{9\pi}{10} - \sin\frac{\pi}{4}\sin\frac{9\pi}{10}\right\}$

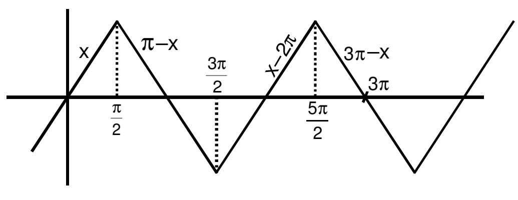
$= y = \cos^{-1}\left\{\cos\left(\frac{\pi}{4} + \frac{9\pi}{10}\right)\right\}$

$= y = \cos^{-1}\left\{\cos\frac{46\pi}{40}\right\} = y = \cos^{-1}\left\{\cos\frac{23\pi}{20}\right\}$

$= y = \cos^{-1}\cos\left\{\pi + \frac{3\pi}{20}\right\} = y = \cos^{-1}\left\{-\left(\cos\frac{3\pi}{20}\right)\right\}$

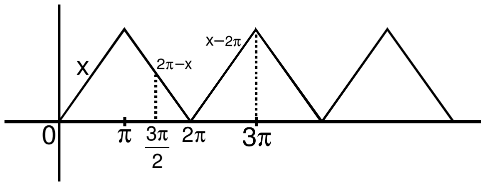
$= y = \pi - \cos^{-1}\left(\cos\frac{3\pi}{20}\right) = \frac{17\pi}{20}$

Sol.5 **(a)** $y = \sin^{-1}(\sin\theta)$, $\theta \in \left[\frac{3\pi}{2}, 3\pi\right]$



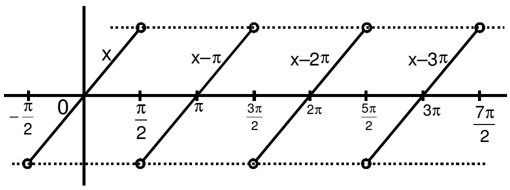
$$y = \sin^{-1}(\sin\theta) = \begin{cases} \theta - 2\pi \ ; & \frac{3\pi}{2} \leq \theta \leq \frac{5\pi}{2} \\ 3\pi - \theta \ ; & \frac{5\pi}{2} < \theta \leq 3\pi \end{cases}$$

(b) $y = \cos^{-1}(\cos\theta)$, $\theta \in \left[\frac{3\pi}{2}, 3\pi\right]$



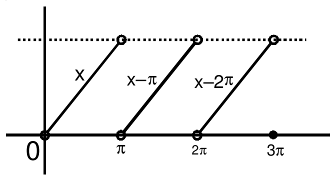
$$y = \cos^{-1}(\cos\theta) = \begin{cases} 2\pi - \theta \ ; & \frac{3\pi}{2} \le \theta \le 2\pi \\ \theta - 2\pi \ ; & 2\pi < \theta \le 3\pi \end{cases}$$

(c) $y = \tan^{-1}(\tan\theta)$



$$y = \tan^{-1}(\tan\theta) = \begin{cases} \theta - 2\pi \ ; & \frac{3\pi}{2} < \theta < \frac{5\pi}{2} \\ \theta - 3\pi \ ; & \frac{5\pi}{2} < \theta \le 3\pi \end{cases}$$

(d) $y = \cot^{-1}(\cot\theta)$



$$y = \cot^{-1}(\text{cof}\,\theta) = \begin{cases} \theta - \pi \ ; & \frac{3\pi}{2} \le \theta < 2\pi \\ \theta - 2\pi \ ; & 2\pi \le \theta < 3\pi \end{cases}$$

Sol.6 **(i)** $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, \ (x > 0)$

$$\tan^{-1}(1) - \tan^{-1}x = \frac{\tan^{-1}x}{2}$$

$$\tan^{-1}(1) = \frac{3}{2}\tan^{-1}x$$

$$\frac{\pi}{4} \times \frac{2}{3} = \tan^{-1}x$$

$$x = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

(ii) $3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{1}{3}\right)$

$$3\tan^{-1}\left(2-\sqrt{3}\right) = \tan^{-1}\left(\frac{x+3}{3x-1}\right)$$

$$3 \times 15º = \tan^{-1}\left(\frac{x+3}{3x-1}\right)$$

$$\tan 45º = \frac{x+3}{3x-1}$$

$$3x - 1 = x + 3$$

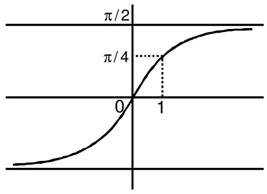
$$2x = 4 \ \Rightarrow \ x = 2$$

Sol.7 $y = \tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}; x > y > 1$

put $x = \tan\alpha$ & $y = \tan\beta$

since $x > y > 1$

so, $\tan\alpha > 1$



$$\frac{\pi}{2} > \alpha > \frac{\pi}{4}$$

Similarly $\frac{\pi}{4} < \beta < \frac{\pi}{2}$

Now, $y = \tan\left\{\frac{1}{2}\sin^{-1}(\sin 2\alpha) + \frac{1}{2}\cos^{-1}(\cos 2\beta)\right\}$

Now $\frac{\pi}{2} < 2\alpha < \pi$ & $\frac{\pi}{2} < 2\beta < \pi$

so, $y = \tan\left\{\frac{1}{2}(\pi - 2\alpha) + \frac{2\beta}{2}\right\}$

$$y = \tan\left\{\frac{\pi}{2} - (\alpha - \beta)\right\} = \cot(\alpha - \beta)$$

$$y = \cot(\tan^{-1}x - \tan^{-1}y)$$

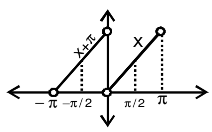
$$y = \cot\left\{\cot^{-1}\frac{1+xy}{x-y}\right\} = \frac{1+xy}{x-y}$$

Sol.8 **(i)** $\tan^{-1}x = -\pi + \cot^{-1}\left(\frac{1}{x}\right), \ x < 0$

put $\tan^{-1}x = \theta \ \Rightarrow \ x = \tan\theta, \ \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$x = \tan\theta \ \Rightarrow \ \frac{1}{x} = \cot\theta$$

$$\cot^{-1}\left(\frac{1}{x}\right) = \cot^{-1}(\cot\theta)$$



so, $\cot^{-1}(\cot\theta) = \begin{cases} \theta+\pi \ ; & -\dfrac{\pi}{2} \le \theta < 0 \\ \theta \ ; & 0 < \theta \le \dfrac{\pi}{2} \end{cases}$

Now given that $x < 0 \Rightarrow \theta < 0$

so $\cot^{-1}(\cot\theta) = \theta + \pi = \tan^{-1} x + \pi$

so, $\cot^{-1}\left(\dfrac{1}{x}\right) = \tan^{-1} x + \pi$

or $\tan^{-1} x = -\pi + \cot^{-1}\left(\dfrac{1}{x}\right)$

consider $y = \sin^{-1}\dfrac{x}{\sqrt{1+x^2}}$ $(x = \tan\theta)$

$\theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \Rightarrow y = \sin^{-1}(\sin\theta) = \theta = \tan^{-1} x$

$(\because \ |\sec\theta| = \sec\theta)$

consider $y = -\cos^{-1}\dfrac{1}{\sqrt{1+x^2}}$

$x = \tan\theta \Rightarrow \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$y = -\cos^{-1}\dfrac{1}{|\sec\theta|}$

$y = -\cos^{-1}(\cos\theta)$

Now, $\cos^{-1}(\cos\theta) = \begin{cases} -\theta \ ; & \left(-\dfrac{\pi}{2} < \theta < 0\right) \\ \theta \ ; & 0 < \theta < \dfrac{\pi}{2} \end{cases}$

but we are given $x < 0$ so,

$\cos^{-1}(\cos\theta) = -\theta \Rightarrow y = -(-\theta)$

$y = \theta = \tan^{-1} x$

(ii) $\cos^{-1} x = \sec^{-1}\dfrac{1}{x}, \ -1 < x < 0$

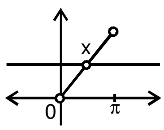
let $\cos^{-1} x = \theta \Rightarrow x = \cos\theta, \ \theta \in [0, \pi]$

$\dfrac{1}{x} = \sec\theta \Rightarrow \sec^{-1}\left(\dfrac{1}{x}\right) = \sec^{-1}(\sec\theta)$

Now $-1 < x < 0 \Rightarrow -1 < \cos\theta < 0$

$1 < \theta < \pi$

$\sec^{-1}\dfrac{1}{x} = \cos^{-1} x$



consider $y = \pi - \sin^{-1}\sqrt{1-x^2}$

put $x = \cos\theta, \ \theta \in [0, \pi]$

$y = \pi - \sin^{-1}|\sin\theta|$

$y = \pi - \sin^{-1}(\sin\theta)$ ......(i)

$-1 < x < 0 \Rightarrow -1 < \cos\theta < 0 \Rightarrow \dfrac{\pi}{2} < \theta < \pi$

$\sin^{-1}\sin\theta = \pi - \theta \quad \left(\dfrac{\pi}{2} < \theta < \pi\right)$

from (1), $y = \pi - (\pi - \theta) = \theta$

$y = \cos^{-1} x$ so, $\cos^{-1} x = (\pi - \sin^{-1}\sqrt{1-x^2})$

consider $y = \pi + \tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x, \ \theta \in [0, \pi]$

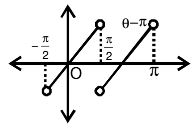
$y = \pi + \tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)$

$y = \pi + \tan^{-1}\tan\theta$

Now, $-1 < x < 0$

$-1 < \cos\theta < 0$

$\dfrac{\pi}{2} < \theta < \pi$



$y = \pi + \theta - \pi = \theta = \cos^{-1} x$

Similarly we can solve for $\cot^{-1}\dfrac{x}{\sqrt{1-x^2}}$

Let $y = \cot^{-1}\dfrac{x}{\sqrt{1-x^2}}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x, \ \theta \in [0, \pi]$

$y = \cot^{-1}\dfrac{\cos\theta}{|\sin\theta|} = \cot^{-1}(\cot\theta)$

$-1 < x < 0$

$-1 < \cos\theta < 0 \Rightarrow \dfrac{\pi}{2} < \theta < \pi$

$y = \theta = \cos^{-1} x$

Sol.9 $\sin^2\{2\cos^{-1}(\tan x)\} = 1$

$\Rightarrow \sin\{2\cos^{-1}(\tan x)\} = \pm 1$

$\sin\{2\cos^{-1}(\tan x)\} = 1$ $\qquad$ $\sin\{2\cos^{-1}(\tan x)\} = -1$

$\sin(2\cos^{-1}(\tan x)) = 1$

$\sin^{-1}\sin(2\cos^{-1}(\tan x)) = \sin^{-1} 1$

$\sin^{-1}\sin(2\cos^{-1}(\tan x)) = \dfrac{\pi}{2}$ ...(1)

$-\dfrac{\pi}{4} \le x \le \dfrac{\pi}{4}$

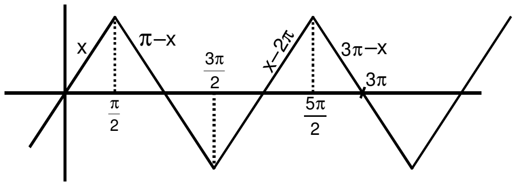
$-1 \le \tan x \le 1$

$0 \le \cos^{-1} (\tan x) \le \pi$

$0 \le \underbrace{2\cos^{-1}(\tan x)}_{m} \le 2\pi$

$0 \le m \le 2\pi$

$\sin^{-1} \sin m = \frac{\pi}{2}$



$2 \cos^{-1} (\tan x) = \frac{\pi}{2} \qquad 0 \le m < \frac{\pi}{2}$

$\cos^{-1} (\tan x) = \frac{\pi}{4}$

$\tan x = \frac{1}{\sqrt{2}} \Rightarrow x = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\pi - 2 \cos^{-1} (\tan x) = \frac{\pi}{2} \qquad \frac{\pi}{2} \le m < \frac{3\pi}{2}$

$2 \cos^{-1} (\tan x) = \frac{\pi}{2} \Rightarrow x = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$2 \cos^{-1} (\tan x) - 2\pi = \frac{\pi}{2} \qquad \frac{3\pi}{2} \le m < 2\pi$

$2 \cos^{-1} (\tan x) = \frac{5\pi}{2} \Rightarrow \cos^{-1} (\tan x) = \frac{5\pi}{4}$

$\tan x = -\frac{1}{\sqrt{2}} \Rightarrow x = \tan^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

similar value of x we will get if we solve
$\sin (2 \cos^{-1} (\tan x)) = 1$
so number of values = 2

Sol.10 $\alpha = 2 \tan^{-1} \left(\frac{1+x}{1-x}\right)$ & $b = \sin^{-1} \left(\frac{1-x^2}{1-x^2}\right)$

for $0 < x < 1$
Put $x = \tan \theta$

$\alpha = 2\tan^{-1} \left(\frac{1+x}{1-x}\right)$

$\alpha = 2\left(\tan^{-1}1 + \tan^{-1} x\right)$

$\alpha = \frac{\pi}{2} + 2\tan^{-1} x$

$\alpha = \frac{\pi}{2} + 2 \tan^{-1} (\tan \theta) \qquad ....(1)$

$\beta = \sin^{-1} (\cos 2\theta)$

$\beta = \sin^{-1} \left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right) \qquad ....(2)$

Now, given, $0 < x < 1$

$\Rightarrow 0 < \tan\theta < 1 \Rightarrow \boxed{0 < \theta < \frac{\pi}{4}}$

Now, $0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} < -2\theta < 0 \Rightarrow \boxed{0 < \frac{\pi}{2} - 2\theta < \frac{\pi}{2}}$

So, $\alpha = \frac{\pi}{2} + 2\theta$

$\beta = \frac{\pi}{2} - 2\theta \Rightarrow \alpha + \beta = \pi$

Sol.11 Given $x = \sin (2\tan^{-1} 2)$

& $y = \sin \left(\frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)\right)$

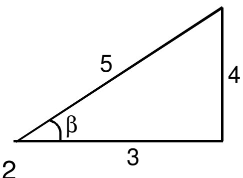
Consider, $x = \sin (2 \tan^{-1} 2)$
Let $\tan^{-1}2 = \alpha \Rightarrow \tan \alpha = 2$
Now, $x = \sin 2\alpha$

$x = \sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha} = \frac{4}{5}$

Now, $y = \sin \left\{\frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)\right\}$

Let $\tan^{-1} \frac{4}{3} = \beta \Rightarrow \tan \beta = \frac{4}{3}$

$\cos \beta = \frac{3}{5}$



$1 - 2 \sin^2 \frac{\beta}{2} = \frac{3}{5}$

$\Rightarrow 2 \sin^2 \frac{\beta}{2} = \frac{2}{5}$

$\Rightarrow \sin \frac{\beta}{2} = \pm \frac{1}{\sqrt{5}} = y$

So, $x = \frac{4}{5}$ & $y = \pm\frac{1}{\sqrt{5}}$

Squaraing in $y = \pm \frac{1}{\sqrt{5}} \Rightarrow y^2 = \frac{1}{5}$

$4y^2 = \frac{4}{5} = x \Rightarrow x = 4y^2$

Sol.12 **(i)** $\underbrace{\cos^{-1}\frac{1}{\sqrt{3}}}_{\theta} - \underbrace{\cos^{-1}\frac{1}{6}}_{\alpha} + \underbrace{\cos^{-1}\left(\frac{\sqrt{10}-1}{3\sqrt{2}}\right)}_{\beta}$

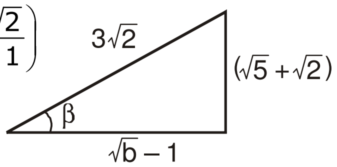
$\cos\theta = \frac{1}{\sqrt{3}}$ $\qquad$ $\cos\alpha = \frac{1}{\sqrt{6}}$

$\tan\theta = \sqrt{2}$ $\qquad$ $\tan\alpha = \sqrt{5}$

$\theta = \tan^{-1}\sqrt{2}$ $\qquad$ $\alpha = \tan^{-1}\sqrt{5}$

$\cos\beta = \frac{\sqrt{10}-1}{3\sqrt{2}} \Rightarrow \tan\beta = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{10}-1}$

$\beta = \tan^{-1}\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{10}-1}\right)$



$= -\tan^{-1}\left(\frac{\sqrt{5}+\sqrt{2}}{1-\sqrt{5}\cdot\sqrt{2}}\right)$

$= -\left(\tan^{-1}\sqrt{5} + \tan^{-1}\sqrt{2} - \pi\right)$

LHS $= \theta - \alpha + \beta$

$= \tan^{-1}\sqrt{2} - \tan^{-1}\sqrt{5}$

$- (\tan^{-1}\sqrt{5} + \tan^{-1}\sqrt{2} - \pi)$

$= \pi - 2\tan^{-1}\sqrt{5}$ = RHS

(ii) $2\tan^{-1}(\text{cosec}\tan^{-1}x - \tan\cot^{-1}x) = \tan^{-1}x$

Let $\tan^{-1}x = \theta \Rightarrow \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, $2\tan^{-1}\left\{\text{cosec}\theta - \tan\left(\frac{\pi}{2} - \tan^{-1}x\right)\right\}$

$\Rightarrow 2\tan^{-1}\{\text{cosec}\theta - \cot\theta\}$

$\Rightarrow 2\tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\} \quad \Rightarrow \quad 2\tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$-\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$

So, $\quad 2\tan^{-1}\left(\tan\frac{\theta}{2}\right) = 2 \cdot \frac{\theta}{2} = \tan^{-1}x$

(iii) $\quad \cos^{-1}\left(\frac{63}{65}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \sin^{-1}\frac{3}{5}$

$\cos^{-1}\frac{63}{65} = \sin^{-1}\frac{3}{5} - 2\tan^{-1}\frac{1}{5}$

$= \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{5}{12} = \tan^{-1}\frac{16}{36}$

$= \cos^{-1}\left(\frac{63}{65}\right)$ $\qquad$ LHS = RHS

Sol.13 **(i)** $\quad y = \tan\left[\cos^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)\right]$

$y = \tan\left[\frac{\pi}{3} + \left(-\frac{\pi}{6}\right)\right]$

$y = \tan\left[\frac{\pi}{6}\right] \quad \Rightarrow \quad \frac{1}{\sqrt{3}}$

(ii) $\quad y = \sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$

$y = \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = \sin\left(\frac{3\pi}{6}\right) = 1$

(iii) $\quad y = \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

$y = \cos^{-1}\cos\left(2\pi - \frac{5\pi}{6}\right)$

$y = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$

(iv) $\quad y = \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$y = \frac{2\pi}{3} - \pi = \frac{-\pi}{3}$

(v) $\quad y = \cos\left(\tan^{-1}\frac{3}{4}\right)$

$y = \cos\left\{\cos^{-1}\left(\frac{4}{5}\right)\right\}$

$y = \frac{4}{5}$

(vi) $\quad y = \tan\left[\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{2}\right]$

$y = \tan\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\frac{2}{3}\right]$

$y = \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{6}{4.3}} = \frac{17}{6}$

Sol.14 **(i)** $\quad y = \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

$$y = \sin\left[\frac{\pi}{2} + \sin^{-1}\frac{\sqrt{3}}{2}\right]$$

$$\Rightarrow \quad y = \sin\left[\frac{\pi}{2} + \frac{\pi}{3}\right]$$

$$\Rightarrow \quad y = \sin\left[\frac{5\pi}{6}\right] = \frac{1}{2}$$

(ii) $y = \cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

$$y = \cos\left[\pi - \frac{\pi}{6} + \frac{\pi}{6}\right] = -1$$

(iii) $y = \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$$y = \frac{3\pi}{4} - \pi = -\frac{\pi}{4}$$

(iv) $y = \cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$

$$y = \cos^{-1}\cos\left(2\pi - \frac{2\pi}{3}\right)$$

$$y = \cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \frac{2\pi}{3}$$

(v) $y = \sin\left[\cos^{-1}\frac{3}{5}\right]$

$$y = \sin^{-1}\left[\sin^{-1}\frac{4}{5}\right] = \frac{4}{5}$$

(vi) $y = \tan^{-1}\left(\frac{3\sin 2\alpha}{5 + 3\cos 2\alpha}\right) + \tan^{-1}\left(\frac{\tan\alpha}{4}\right)$

$$= \tan^{-1}\left(\frac{6\tan\alpha}{8 + 2\tan^2\alpha}\right) + \tan^{-1}\left(\frac{\tan\alpha}{4}\right)$$

$$= \tan^{-1}\left(\frac{3\tan\alpha}{4 + \tan^2\alpha}\right) + \tan^{-1}\left(\frac{\tan\alpha}{4}\right)$$

$$= \tan^{-1}(\tan\alpha) = \alpha$$

Sol.15 (i) LHS = $\underbrace{2\cos^{-1}\frac{3}{\sqrt{3}}}_{\theta} + \cot^{-1}\frac{16}{63} + \underbrace{\frac{1}{2}\cos^{-1}\frac{7}{25}}_{\alpha}$

$$= 2\theta + \tan^{-1}\frac{63}{16} + \frac{\alpha}{2}$$

$\cos\theta = \frac{3}{\sqrt{13}}$ $\qquad \alpha = \cos^{-1}\frac{7}{25}$

$\Rightarrow \tan\theta = \frac{2}{3}$; $\cos\alpha = \frac{7}{25} = 1 - \sin^2\frac{\alpha}{2}$

$\tan 2\theta = \frac{12}{5}$; $\sin^2\frac{\alpha}{2} = \frac{9}{25}$

$2\theta = \tan^{-1}\frac{12}{5}$; $\sin\frac{\alpha}{2} = \frac{3}{5} \Rightarrow \tan\frac{\alpha}{2} = \frac{3}{4}$

LHS $= \tan^{-1}\frac{12}{5} + \tan^{-1}\frac{63}{16} + \tan^{-1}\frac{3}{4}$

$$= \tan^{-1}\frac{12}{5} + \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{63}{16}$$

$$= \pi - \tan^{-1}\left(\frac{\frac{12}{5} + \frac{3}{4}}{1 - \frac{36}{20}}\right) + \tan^{-1}\frac{63}{16} = \pi = \text{RHS}$$

(ii) LHS : $\cos^{-1}\left(\frac{5}{13}\right) + \pi - \cos^{-1}\left(\frac{7}{25}\right) + \sin^{-1}\left(\frac{36}{325}\right)$

$$\cos^{-1}\frac{5}{13} + \sqrt{2} - \cos^{-1}\left(\frac{7}{25}\right) + \sin^{-1}\left(\frac{36}{325}\right)$$

$$\tan^{-1}\left(\frac{12}{5}\right) - \tan^{-1}\frac{24}{7} + \tan\left(\frac{36}{323}\right) + \pi$$

$$\tan^{-1}\left(\frac{-36}{323}\right) + \tan^{-1}\left(\frac{36}{323}\right) + \pi$$

$= \pi = \text{RHS}$

(iii) LHS : $\cos^{-1}\sqrt{\frac{2}{3}} - \cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}}$

LHS $\quad \tan^{-1}\frac{1}{\sqrt{2}} - \tan^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}+1}\right)$

$$\tan^{-1}\frac{1}{\sqrt{2}} - \tan^{-1}\sqrt{3} + \tan^{-1}\sqrt{2}$$

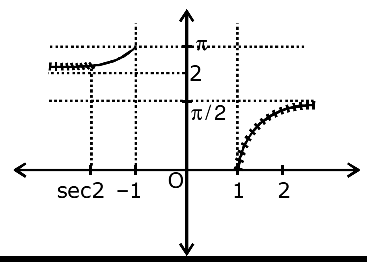
$$\frac{\pi}{2} - \tan^{-1}\sqrt{3} = \cot^{-1}\sqrt{3} = \frac{\pi}{6}$$

(iv) $(\sec^{-1}x)^2 - 6(\sec^{-1}x) + 8 > 0$

$(\sec^{-1}x)^2 - 4\sec^{-1}x - 2\sec^{-1}x + 8 > 0$

$(\sec^{-1}x - 2)(\sec^{-1}x - 4) > 0$

$\sec^{-1}x < 2$ or $\sec^{-1}x > 4$



$x \in (-\infty, \sec 2] \cup [1, \infty)$

Sol.16 (i) $f(x) = \cos^{-1}\left(\dfrac{2x}{x+1}\right)$

$-1 \le \dfrac{2x}{x+1} \le 1 \Rightarrow \dfrac{2x}{x+1} \ge -1$ & $\dfrac{2x}{x+1} \le 1$

$\Rightarrow \dfrac{3x+1}{x+1} \ge 0$ & $\dfrac{x-1}{x+1} \le 0$

$\Rightarrow x \in (-\infty, -1) \cup \left[\dfrac{1}{3}, \infty\right)$ & $x \in (-1, 1]$

taking intersection, $x \in \left[-\dfrac{1}{3}, 1\right]$

(ii) $\cos(\sin x) \ge 0$; $-1 \le \dfrac{1+x^2}{2x} \le 1$

$x \in R$ $\Rightarrow x > 0 \cup \{-1\}$

$\Rightarrow x < 0 \cup \{1\}$

Intersection $x \in \{-1, 1\}$

(iii) $f(x) = \sin^{-1}\left(\dfrac{x-3}{2}\right) - \log_{10}(4-x)$

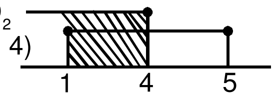
D_1 : $-1 \le \dfrac{x-3}{2} \le 1$

$-2 \le x - 3 \le 2 \Rightarrow 1 \le x \le 5$

D_2 : $4 - x > 0 \Rightarrow x < 4$

$D_1 \cap D_2$

$x \in [1, 4)$



(iv) $f(x) = \dfrac{\sqrt{1-\sin x}}{\log_5(1-4x^2)} + \cos^{-1}(1 - \{x\})$

$D_1 : 1 - \sin x \ge 0 \Rightarrow \sin x \le 1$

$\Rightarrow \quad x \le \dfrac{\pi}{2}$

$D_2 : 1 - 4x^2 > 0 \Rightarrow \quad 4x^2 - 1 < 0$

$x \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ & $1 - 4x^2 \ne 1$

$(\because \ x \ne 0)$

$D_3 : -1 \le 1 - \{x\} \le 1$

$\Rightarrow \quad -2 \le -\{x\} \le 0$

$\Rightarrow \quad 0 \le \{x\} \le 2 \Rightarrow x \in R$

$D_1 \cap D_2 \cap D_3 \Rightarrow x \in \left(-\dfrac{1}{2}, 0\right) \cup \left(0, \dfrac{1}{2}\right)$

(v) $f(x) = \sqrt{3-x} + \cos^{-1}\left(\dfrac{3-2x}{5}\right) + \log_6(2|x|-3)$

$+ \sin^{-1}(\log_2 x)$

$D_1 : 3 - x \ge 0 \quad \Rightarrow \quad x \le 3$

$D_2 : -1 \le \dfrac{3-2x}{5} \le 1 \Rightarrow -1 \le x \le 4$

$D_3 : 2|x| - 3 \ge 0 \quad \Rightarrow x < -\dfrac{3}{2}$ or $x > \dfrac{3}{2}$

$D_4 : -1 \le \log_2 x \le 1 \quad \Rightarrow \dfrac{1}{2} \le x \le 2$

$D_1 \cap D_2 \cap D_3 \cap D_4 \Rightarrow x \in \left(\dfrac{3}{2}, 2\right]$

(vi) $1 - \log_7(x^2 - 5x + 3) > 0$

$\Rightarrow x \in (2, 3)$

$x^2 - 5x + 13 > 0 \Rightarrow \quad x \in R$

$-1 \le \dfrac{3}{2+\sin\theta} \le 1$ where $\theta = \dfrac{9\pi x}{2}$

$\dfrac{3}{2+\sin\theta} > -1 \qquad \dfrac{3}{2+\sin\theta} \le 1$

$\sin\theta \ge -5$ always $\qquad \sin\theta = 1 \Rightarrow \sin\dfrac{9\pi x}{2} = 1$

$\dfrac{9\pi x}{2} = (4n+1)\dfrac{\pi}{2}$

$x = \dfrac{4n+1}{9}$

$n = 5, x = 7/3$

$n = 6, x = 25/9$

(vii) $-1 \le \dfrac{x}{2} \le 1 \quad \Rightarrow x \in [-2, 2]$

for $\tan^{-1}\left[\dfrac{x}{2} - 1\right] \Rightarrow x \in R$

for $\ell n\sqrt{\{x\}} \quad \Rightarrow \{x\} \ne 0 \Rightarrow x \ne I$

finally $x \in (-2, 2) - \{-1, 0, 1\}$

(viii) $\sin(\cos x) \ge 0$

$\downarrow$

$[0, 1]$

$x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] \Rightarrow x \in \left[2n\pi - \dfrac{\pi}{2}, 2n\pi + \dfrac{\pi}{2}\right]$

$-2\cos^2 x + 3\cos x + 1 > 0 \Rightarrow \dfrac{3-\sqrt{17}}{2} < \cos x \le 1$

$-1 \le \dfrac{2\sin x+1}{2\sqrt{2\sin x}} \le 1$

by solving $\{x \mid x = 2n\pi + \dfrac{\pi}{6},\ n \in I\}$

Sol.17 $3\cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\,(4x^2-1)\right)$

Let $\cos^{-1}x = \theta \Rightarrow \quad x = \cos\theta \Rightarrow \theta \in [0, \pi]$

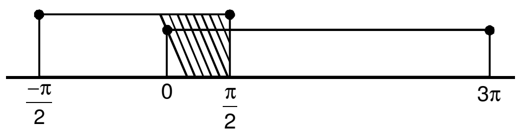
$\sin^{-1}\{|\sin\theta|\,(4\cos^2\theta - 1)\}$

$= \sin^{-1}\{\sin\theta\,(4 - 1 - 4\sin^2\theta)\}$

$\Rightarrow \quad \sin^{-1}\{\sin 3\theta\} = 3\theta \ne$ when

$-\dfrac{-\pi}{2} \le 3\theta \le \dfrac{\pi}{2}$ but we had

$0 \le \theta \le \pi \quad \Rightarrow \quad 0 \le 3\theta \le 3\pi$



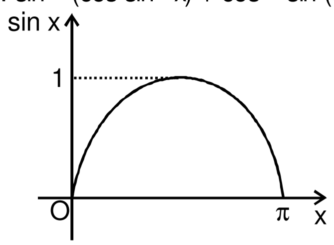
So, $\quad 0 \le 3\theta \le \dfrac{\pi}{2} \Rightarrow 0 \le \theta \le \dfrac{\pi}{6}$

$0 \le \cos^{-1}x \le \dfrac{\pi}{6}$

$\cos\dfrac{\pi}{6} \le x \le \cos 0$

$\dfrac{\sqrt{3}}{2} \le x \le 1 \quad \Rightarrow \quad x \in \left[\dfrac{\sqrt{3}}{2}, 1\right]$

Sol.18 **(i)** LHS : $\sin^{-1}(\cos\sin^{-1}x) + \cos^{-1}\sin(\cos^{-1}x)$



$\sin^{-1}\left\{\cos\left(\dfrac{\pi}{2} - \cos^{-1}x\right)\right\} + \cos^{-1}\{\sin(\cos^{-1}x)\}$

$\sin^{-1}\{\sin(\cos^{-1}x)\} + \cos^{-1}\{\sin(\cos^{-1}x)\}$

Now, $-1 \le x \le 1 \Rightarrow 0 \le \cos^{-1}x \le \pi$

$\sin(\cos^{-1}x)$ is the value from 0 to 1 & then 1 to 0

Hence LHS = $\dfrac{\pi}{2}$

(ii) LHS : $2\tan^{-1}(\text{cosec}\tan^{-1}x - \tan\cot^{-1}x)$

Let $\tan^{-1}x = A \Rightarrow -\dfrac{\pi}{2} < A < \dfrac{\pi}{2}$

LHS

$2\tan^{-1}\left(\text{cosec}\,A - \tan\left(\dfrac{\pi}{2} - A\right)\right)$

$2\tan^{-1}(\text{cosec}\,A - \cot A)$

$\Rightarrow 2\tan^{-1}\left(\dfrac{1-\cos A}{\sin A}\right) = 2\tan^{-1}\left(\tan\dfrac{A}{2}\right)$

$\Rightarrow 2\tan^{-1}\left(\dfrac{2\sin^2\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}\right) - \dfrac{\pi}{4} < \dfrac{A}{2} < \dfrac{\pi}{4}$

So, $\quad 2 \times \dfrac{A}{2} = A = \tan^{-1}x$

(iii) $\tan^{-1}\dfrac{2mn}{(m^2-n^2)} + \tan^{-1}\dfrac{2pq}{(p^2-q^2)}$

$= \tan^{-1}\dfrac{2MN}{(M^2-N^2)}$

$\tan^{-1}\left\{\dfrac{2\left(\frac{n}{m}\right)}{1-\left(\frac{n}{m}\right)^2}\right\} + \tan^{-1}\left\{\dfrac{2\left(\frac{q}{p}\right)}{1-\left(\frac{q}{p}\right)^2}\right\} = \tan^{-1}\left\{\dfrac{2\left(\frac{N}{M}\right)}{1-\left(\frac{N}{M}\right)^2}\right\}$

Now, $y = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, $|x| < 1$

$-1 < x < 1$

as $x = \tan\theta$, so $-1 < \tan\theta < 1$

$-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$

So, $\quad \tan^{-1}(\tan 2\theta) = 2\tan^{-1}x$

So, $\quad 2\tan^{-1}\dfrac{n}{m} + 2\tan^{-1}\dfrac{q}{p} = 2\tan^{-1}\dfrac{N}{M}$

LHS $\dfrac{np+mq}{mp-nq} = \dfrac{N}{M}$

(iv) LHS $\tan\left(\dfrac{\pi}{2} - \cot^{-1}x + \dfrac{\pi}{2} - \cot^{-1}y + \dfrac{\pi}{2} - \cot^{-1}z\right)$

$\tan\left(\dfrac{3\pi}{2} - (\cot^{-1}x + \cot^{-1}y + \cot^{-1}z)\right)$

$\cot(\cot^{-1}x + \cot^{-1}y + \cot^{-1}z)$ = RHS

Sol.19 **(i)** $\quad \sin^{-1}x + \sin^{-1}2x = \dfrac{\pi}{3}$

$\sin^{-1}2x = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) - \sin^{-1}x$

$$\sin^{-1} 2x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\sqrt{1-x^2} - x\times\frac{1}{2}\right)$$

$$4x = \sqrt{3}\ (\sqrt{1-x^2}\ ) - x$$

$$5x = \sqrt{3}\ \sqrt{1-x^2}$$

$$25x^2 = 3 - 3x^2$$

$$x^2 = \frac{3}{28} \quad \Rightarrow \quad x = \frac{1}{2}\sqrt{\frac{3}{7}} \qquad \text{Negative NP}$$

(ii) $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$$\tan^{-1}\left\{\frac{\frac{1}{2x+1}+\frac{1}{4x+1}}{1-\frac{1}{(2x+1)(4x+1)}}\right\} = \tan^{-1}\frac{2}{x^2}$$

$$\frac{4x+1+2x+1}{(2x+1)(4x+1)-1} = \frac{2}{x^2}$$

$$\frac{6x+2}{(8x^2+6x)} = \frac{2}{x^2} \quad \Rightarrow \ 6x^3 + 2x^2 = 16x^2 + 12x$$

$$\Rightarrow \ 6x^3 - 14x^2 - 12x = 0$$

$$\Rightarrow \ x(6x^2 - 14x - 12) = 0$$

(iii) $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$

$\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1} 3x - \tan^{-1} x$

$$\tan^{-1}\left(\frac{x-1+x+1}{1-x^2+1}\right) = \tan^{-1}\left(\frac{3x-x}{1+3x^2}\right)$$

$$\tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{2x}{1+3x^2}\right)$$

$$\frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$$

$x = 0$ $\qquad\qquad$ $1 + 3x^2 = 2 - x^2$

$2x^2 = 1$

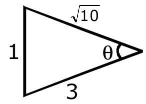
$$x = \pm\frac{1}{2}$$

(iv) $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1} x = \frac{\pi}{4}$

$$\cos^{-1} x = \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}\frac{1}{\sqrt{5}}$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\ \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{2}}\right)$$

$$\cos^{-1} x = \sin^{-1}\left(\frac{1}{\sqrt{10}}\right)$$



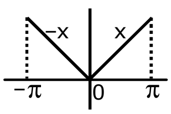
$$\cos^{-1} x = \cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$$

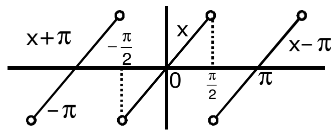
$$x = \frac{3}{\sqrt{10}}$$

(v) $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$

LHS $\quad \pi - \cos^{-1}\frac{1-x^2}{1+x^2} - \tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$

$2\theta \in (-\pi, \pi) \qquad x = \tan\theta \qquad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$





C–1 $\quad -\pi < 2\theta < -\frac{\pi}{2}$

$$\pi - (-2\theta) - (2\theta + \pi) = \frac{2\pi}{3}$$

$$0 = \frac{2\pi}{3}$$

C–2 $\quad -\frac{\pi}{2} < 2\theta < 0$ $\qquad$ **C–3** $\quad 0 < 2\theta < \frac{\pi}{2}$

$\pi - (-2\theta) - 2\theta = \frac{2\pi}{3}$ $\qquad$ $\pi - (2\theta) - 2\theta = \frac{2\pi}{3}$

$\pi = \frac{2\pi}{3}$ $\qquad\qquad$ $\theta = \frac{\pi}{12}$

$$x = \tan\theta = \tan\frac{\pi}{12} = 2 - \sqrt{3}$$

C–4 $\quad \frac{\pi}{2} < 2\theta < \pi$

$$\pi - (2\theta) - (2\theta - \pi) = \frac{2\pi}{3}$$

$2\pi - 4\theta = \frac{2\pi}{3} \Rightarrow \frac{4\pi}{3} = 4\theta \Rightarrow \theta = \frac{\pi}{3}$

$x = \tan\theta = \tan 60^\circ = \sqrt{3}$

(vi) $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ ....(1)

$\cos^{-1}x - \cos^{-1}y = \frac{\pi}{3}$

$\frac{\pi}{2} - \sin^{-1}x - \left(\frac{\pi}{2} - \sin^{-1}y\right) = \frac{\pi}{3}$

$\sin^{-1}y - \sin^{-1}x = \frac{\pi}{3}$ ....(2)

(1) + (2)

$2\sin^{-1}y = \pi$

$\sin^{-1}y = \frac{\pi}{2} \Rightarrow y = 1$

and by puting y value we will get $x = \frac{1}{2}$

(vii) $2\tan^{-1}x = \cos^{-1}\frac{1-a^2}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2}$

put $a = \tan\theta_1$

$\Rightarrow \theta_1 = \tan^{-1}a \quad \theta_1 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cos^{-1}\frac{1-\tan^2\theta_1}{1+\tan^2\theta_1} = \cos^{-1}(\cos 2\theta_1) \quad 2\theta, (-\pi, \pi)$

$= 2\theta_1$

$2\theta, (0, \pi)$

$= 2\tan^{-1}a$ become $a > 0$

similarly

$\cos^{-1}\frac{1-b^2}{1+b^2} = 2\tan^{-1}b$

$2\tan^{-1}x = 2\tan^{-1}a - 2\tan^{-1}b$

$\tan^{-1}x = \tan^{-1}\frac{a-b}{1+ab}$

$x = \frac{a-b}{1+ab}$

Sol.20 $\sin^{-1}(\sin 8) = \sin^{-1}(\sin(3\pi - 8)) = 3\pi - 8$

$\tan^{-1}(\tan 10) = \tan^{-1}(\tan(10 - 3\pi)) = 10 - 3\pi$

$\cos^{-1}(\cos 12) = \cos^{-1}(\cos(4\pi - 12)) = 4\pi - 12$

$\sec^{-1}(\sec 9) = \sec^{-1}(\sec(9 - 2\pi)) = 9 - 2\pi$

$\cot^{-1}(\cot 6) = \cot^{-1}(\cot(6 - \pi)) = 6 - \pi$

$\operatorname{cosec}^{-1}(\operatorname{cosec} 7) = \operatorname{cosec}^{-1}(\operatorname{cosec}(7 - 2\pi)) = 7 - 2\pi$

$y = (3\pi - 8) + (3\pi - 10) + (4\pi - 12) + (2\pi - 9) + (-\pi + 6) + (2\pi - 7)$

$y = 13\pi - 40$

$\Rightarrow a = 13$ and $b = -40 \Rightarrow a - b = 13 - (-40) = 53$

Sol.21 $T_1 = \sin^{-1}\sin\left(5\pi - \frac{2\pi}{7}\right) = \sin^{-1}\sin\left(\frac{2\pi}{7}\right) = \frac{2\pi}{7}$

$T_2 = \cos^{-1}\cos\left(6\pi + \frac{4\pi}{7}\right) = \cos^{-1}\cos\frac{4\pi}{7} = \frac{4\pi}{7}$

$T_3 = \tan^{-1}\left\{-\tan\left(2\pi - \frac{3\pi}{8}\right)\right\} = \tan^{-1}\tan\frac{3\pi}{8} = \frac{3\pi}{8}$

$T_4 = \pi - \cot^{-1}\cot\left(\frac{19\pi}{8}\right) = -\pi - \cot^{-1}\left(2\pi + \frac{3\pi}{8}\right)$

$= \pi - \frac{3\pi}{8} = \frac{5\pi}{8}$

$S = \frac{2\pi}{7} + \frac{4\pi}{7} + \frac{3\pi}{8} + \frac{5\pi}{8} = \frac{6\pi}{7} + \pi = \frac{13\pi}{7}$

Sol.22 $u = \cot^{-1}\sqrt{\cos 2\theta} - \tan^{-1}\sqrt{\cos 2\theta}$

$u = \frac{\pi}{2} - \tan^{-1}\sqrt{\cos 2\theta} - \tan^{-1}\sqrt{\cos 2\theta}$

$2\tan^{-1}\sqrt{\cos 2\theta} = \frac{\pi}{2} - u$

$\tan^{-1}\left(\frac{2\sqrt{\cos 2\theta}}{1 - \cos 2\theta}\right) = \frac{\pi}{2} - u$

$\cos\left(\tan^{-1}\left(\frac{2\sqrt{\cos 2\theta}}{1 - \cos 2\theta}\right)\right) = \sin u$

Let $\tan^{-1}\frac{2\sqrt{\cos 2\theta}}{1 - \cos 2\theta} = \theta$

$\tan\theta = \frac{2\sqrt{\cos 2\theta}}{1 - \cos 2\theta}$

$\cos\theta = \sin u$

$\frac{1}{\sqrt{1 + \frac{4\cos 2\theta}{(1 - \cos 2\theta)^2}}} = \sin u$

$\frac{1-\cos 2\theta}{1+\cos 2\theta} = \sin u \Rightarrow \tan^2\theta = \sin u$

Sol.23 $f(x) = \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$

$= \frac{\pi}{2} - \cos^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x)$

$= \frac{\pi}{2} - \pi + \cos^{-1}(4x^3 - 3x) + \cos^{-1}(4x^3 - 3x)$

$= -\frac{\pi}{2} + 2\cos^{-1}(4x^3 - 3x)$

put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x \quad \theta \in [0, \pi]$

$= -\frac{\pi}{2} + 2\cos^{-1}(\cos 3\theta)$

$-1 \le x \le -\frac{1}{2} \Rightarrow -1 \le \cos\theta \le -\frac{1}{2}$

$\frac{2\pi}{3} \le \theta \le \pi \Rightarrow 2\pi \le 3\theta \le 3\pi$

$= -\frac{\pi}{2} + 2(3\theta - 2\pi)$

$= -\frac{\pi}{2} + 6\theta - 4\pi$

$= -\frac{9\pi}{2} + 6\cos^{-1}x.$

$a = 6, b = -\frac{9}{2}$

Sol.24 **(i)** $T_n = \sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{\frac{n}{n+1}} - \sqrt{\frac{n-1}{n}}\frac{1}{\sqrt{n+1}}\right]$

$= \sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{1-\frac{1}{n+1}} - \sqrt{1-\frac{1}{n}}\frac{1}{\sqrt{n+1}}\right]$

If $\sin\theta = \frac{1}{\sqrt{n}}$ then $\cos\theta = \sqrt{1-\frac{1}{n}}$

& If $\sin\phi = \frac{1}{\sqrt{n+1}}$ then $\cos\phi = \sqrt{1-\frac{1}{n+1}}$

$T_n = \sin^{-1}(\sin\theta\cos\phi - \cos\theta\sin\phi)$

$= \sin^{-1}(\sin(\theta - \phi))$

$T_n = \theta - \phi = \sin^{-1}\frac{1}{\sqrt{n}} - \sin^{-1}\frac{1}{\sqrt{n+1}}$

$\Rightarrow \quad S_n = \sin^{-1}1 - \sin^{-1}\frac{1}{\sqrt{n+1}}$

$S_\infty = \sin^{-1}1 = \frac{\pi}{2}$

(ii) $T_n = \tan^{-1}\frac{2^{n-1}(2-1)}{1+2^n\cdot 2^{n-1}}$

$= \tan^{-1}2^n - \tan^{-1}2^{n-1}$

$S_n = \tan^{-1}2^n - \tan^{-1}1$

$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

(iii) $S_n = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{21} + \ldots\ldots$

$T_n = \tan^{-1}\frac{1}{1+n+n^2}$

$= \tan^{-1}\frac{n+1-n}{1+n(n+1)}$

$T_n = \tan^{-1}(n+1) - \tan^{-1}n$

$S_n = \tan^{-1}(n+1) - \tan^{-1}1$

$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

(iv) $y = \tan^{-1}(x+1) - \tan^{-1}x$

$+ \tan^{-1}(x+2) - \tan^{-1}(x+1)$

$+ \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$

$y = \tan^{-1}(x+n) - \tan^{-1}x$

(v) $T_n = \tan^{-1}\frac{1}{2\cdot n^2} = \tan^{-1}\frac{2}{4n^2}$

$= \tan^{-1}\frac{2}{1+4n^2-1}$

$= \tan^{-1}\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}$

$T_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$S_n = \tan^{-1}(2n+1) - \tan^{-1}1$

$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Sol.25 **(i)** $\cot^{-1}x + \cot^{-1}(n^2 - x + 1) = \cot^{-1}(n-1)$

$\tan^{-1}\frac{1}{n-1} = \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{n^2-x+1}$

$= \tan^{-1}\left[\frac{\frac{1}{x} + \frac{1}{n^2-x+1}}{1 - \frac{1}{x(n^2-x+1)}}\right]$

$\frac{1}{n-1} = \frac{n^2+1}{(n^2+1)x - x^2 + 1}$

$n^2 - n(x^2+1) + 1 + (x-1)(x^2+1) = 0$

$(x^2 - n^2) - (n^2+1)(x-n) = 0$

$(x - n)(x + n - n^2 - 1) = 0$
$x = n$ or $n^2 - n + 1$

(ii) $\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1} b - \sec^{-1} a$

$\cos^{-1}\frac{a}{x} + \cos^{-1}\frac{1}{a} = \cos^{-1}\frac{b}{x} + \cos^{-1}\frac{1}{b}$

$$\cos^{-1}\left[\frac{a}{x}.\frac{1}{a} - \sqrt{\left(1-\frac{a^2}{x^2}\right)\left(1-\frac{1}{a^2}\right)}\right]$$

$$= \cos^{-1}\left[\frac{b}{x}.\frac{1}{b} - \sqrt{\left(1-\frac{1}{b^2}\right)\left(1-\frac{b^2}{x^2}\right)}\right]$$

$$\frac{1}{x} - \frac{\sqrt{x^2-a^2}\ \sqrt{a^2-1}}{ax} = \frac{1}{x} - \frac{\sqrt{b^2-1}\ \sqrt{x^2-b^2}}{bx}$$

$b^2(a^2 - 1)(x^2 - a^2) = a^2(b^2 - 1)(x^2 - b^2)$
$x^2(a^2b^2 - b^2 - a^2b^2 + a^2) = a^2b^2(a^2 - 1 - b^2 + 1)$
$x^2(a^2 - b^2) = a^2b^2(a^2 - b^2) \Rightarrow x = ab$

(iii) $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$

$$\tan^{-1}\left[\frac{\frac{x-1}{x+1}+\frac{2x-1}{2x+1}}{1-\frac{(x-1)(2x-1)}{(x+1)(2x+1)}}\right] = \tan^{-1}\frac{23}{36}$$

$x = \frac{4}{3}$

Sol.26 (i) $(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$
$t^2 - 5t + 6 > 0$
$t < 2$ or $t > 3$
$\cot^{-1} x < 2$ $\quad$ $\cot^{-1} x > 3$
$x > \cot 2$ $\quad$ $x < \cot 3$

(ii) $\sin^{-1} x > \cos^{-1} x$

$\sin^{-1} x > \frac{\pi}{2} - \sin^{-1} x$

$\frac{\pi}{2} \geq \sin^{-1} x > \frac{\pi}{4}$

$1 \geq x > \frac{1}{\sqrt{2}}$

(iii) $\tan^2(\sin^{-1} x) > 1$
$\tan(\sin^{-1} x) > 1$ or $\tan(\sin^{-1} x) < -1$

$\frac{\pi}{2} > \sin^{-1} x > \frac{\pi}{4}$ $\qquad$ $-\frac{\pi}{4} < \sin^{-1} x < -\frac{\pi}{2}$

$1 > x > \frac{1}{\sqrt{2}}$

$-1 > x > -\frac{1}{\sqrt{2}}$

$x \in \left(\frac{1}{\sqrt{2}}, 1\right) \cup \left(-1, -\frac{1}{\sqrt{2}}\right)$

EXERCISE – IV HINTS & SOLUTIONS

Sol.1 (a) $\tan\left[\frac{x}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left(\frac{x}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

$$= \frac{1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} + \frac{1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}$$

$$= \frac{\left(1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right)^2 + \left(1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)\right)^2}{1-\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}$$

$$= 2\left(\frac{1+\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1-\tan^2\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}\right)$$

$$= \frac{2}{\cos\left(2\times\frac{1}{2}\cos^{-1}\frac{a}{b}\right)} = \frac{2}{\cos\left(\cos^{-1}\frac{a}{b}\right)} = \frac{2b}{a}$$

(b) Let $\cos^{-1}\left(\frac{\cos x+\cos y}{1+\cos x\cos y}\right) = \theta$

$$\cos\theta = \frac{\cos x+\cos y}{1+\cos x\cos y}$$

$$\frac{1}{\cos\theta} = \frac{1+\cos x\cos y}{\cos x+\cos y}$$

$$\frac{1-\cos\theta}{1+\cos\theta} = \frac{1+\cos x\cos y-\cos x-\cos y}{1+\cos x\cos y+\cos x+\cos y}$$

$$\tan^2\frac{\theta}{2} = \frac{(1-\cos x)(1-\cos y)}{(1+\cos x)(1+\cos y)}$$

$$\frac{\tan^2\theta}{2} = \tan^2\frac{x}{2}\ \tan^2\frac{y}{2}$$

$$\tan\frac{\theta}{2} = \tan\frac{x}{2}\ \tan\frac{y}{2}$$

$$\frac{\theta}{2} = \tan^{-1}\left(\frac{\tan x}{2}\frac{\tan y}{2}\right)$$

$$\theta = 2\tan^{-1}\left(\frac{\tan x}{2}\frac{\tan y}{2}\right)$$

(c) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right)$$

$$= \cos^{-1}\left(\frac{1-\left(\frac{a-b}{a+b}\right)\tan^2\frac{x}{2}}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{x}{2}}\right)$$

$$= \cos^{-1}\left(\frac{(a+b)-(a-b)\tan^2\frac{x}{2}}{(a+b)+(a-b)\left(\tan^2\frac{x}{2}\right)}\right)$$

$$= \cos^{-1}\left[\frac{a\left(1-\tan^2\frac{x}{2}\right)+b\left(1+\tan^2\frac{x}{2}\right)}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}\right]$$

$$= \cos^{-1}\left[\frac{a\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)+b}{a+b\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}\right] = \cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$$

Sol.2 $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

$$\tan y = \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$$

$$\frac{\sin y}{\cos y} = \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$$

$$\frac{\sin y+\cos y}{\sin y-\cos y} = \frac{2\sqrt{1+x^2}}{-2\sqrt{1-x^2}}$$

Square on both side $\frac{1+\sin 2y}{1-\sin 2y} = \frac{1+x^2}{1-x^2}$

$(1 - x^2)(1 + \sin 2y) = (1 + x^2)(1 - \sin 2y)$

$1 + \sin 2y - x^2 - x^2 \sin 2y = 1 - \sin 2y + x^2 - x^2 \sin y$

$2\sin 2y = 2x^2 \qquad x^2 = \sin 2y$

Sol.3 (a) $S_n = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{21} + ...$

$T_n = \tan^{-1}\frac{1}{1+n+n^2}$

$= \tan^{-1}\frac{n+1-n}{1+n(n+1)}$

$T_n = \tan^{-1}(n+1) - \tan^{-1} n$

$S_n = \tan^{-1}(n+1) - \tan^{-1} 1$

$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

(b) $T_n = \tan^{-1}\frac{2^{n-1}(2-1)}{1+2^n \cdot 2^{n-1}}$

$= \tan^{-1} 2^n - \tan^{-1} 2^{n-1}$

$S_n = \tan^{-1} 2^n - \tan^{-1} 1$

$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

(c) $y = \tan^{-1}(x+1) - \tan^{-1} x$

$+ \tan^{-1}(x+2) - \tan^{-1}(x+1)$

$+ \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$

$y = \tan^{-1}(x+n) - \tan^{-1} x$

(d) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} + ...$

$T_n = \tan^{-1}\frac{1}{2n^2} = \tan^{-1}\frac{2}{4n^2} = \tan^{-1}\frac{2}{1+4n^2-1}$

$= \tan^{-1}\left[\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right] = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$S_n = \tan^{-1}(2n+1) - \tan^{-1} 1$

$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

(e) $T_n = \sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{\frac{n}{n+1}} - \sqrt{\frac{n-1}{n}}\frac{1}{\sqrt{n+1}}\right]$

$= \sin^{-1}\left[\frac{1}{\sqrt{n}}\sqrt{1-\frac{1}{n+1}} - \sqrt{1-\frac{1}{n}}\frac{1}{\sqrt{n+1}}\right]$

If $\sin\theta = \frac{1}{\sqrt{n}}$ then $\cos\theta = \sqrt{1-\frac{1}{n}}$

& If $\sin\phi = \frac{1}{\sqrt{n+1}}$ then $\cos\phi = \sqrt{1-\frac{1}{n+1}}$

$T_n = \sin^{-1}(\sin\theta\cos\phi - \cos\theta\sin\phi)$

$= \sin^{-1}(\sin(\theta - \phi))$

$T_n = \theta - \phi = \sin^{-1}\frac{1}{\sqrt{n}} - \sin^{-1}\frac{1}{\sqrt{n+1}}$

$\Rightarrow \quad S_n = \sin^{-1} 1 - \sin^{-1}\frac{1}{\sqrt{n+1}}$

$S_\infty = \sin^{-1} 1 = \frac{\pi}{2}$

Sol.4 $\frac{\beta^3}{2}\operatorname{cosec}^2\left[\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right] \qquad \cot 2\theta = \frac{\alpha}{\beta}$

$\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha} = \theta \Rightarrow \tan^{-1}\frac{\beta}{\alpha} = 2\theta \Rightarrow \tan 2\theta = \frac{\beta}{\alpha}$

$\frac{\beta^3}{2}\operatorname{cosec}^2\theta$; $\cos 2\theta = 1 - 2\sin^2\theta$

$\frac{\beta^3}{1-\cos^2\theta} + \frac{\alpha^3}{2}\sec^2\left(\frac{1}{2}.\tan^{-1}\frac{\alpha}{\beta}\right)$

$\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta} = \phi \Rightarrow \tan^{-1}\frac{\alpha}{\beta} = 2\phi$

$\tan 2\phi = \frac{\alpha}{\beta}$

$\frac{\alpha^3}{2\cos^2\phi} \qquad 2\cos^2\theta - 1 = \cos, 1 + \cos\theta$

$\frac{\beta^3}{1-\cos 2\theta} + \frac{\alpha^3}{1+\cos 2\phi}$

$\frac{\beta^3}{1-\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}} + \frac{\alpha^3}{1+\frac{\beta}{\sqrt{\alpha^2+\beta^2}}}$

$\frac{\beta^3(\sqrt{\alpha^2+\beta^2})}{\sqrt{\alpha^2+\beta^2}-\alpha} + \frac{\alpha^3(\sqrt{\alpha^2+\beta^2})}{\sqrt{\alpha^2+\beta^2}+\beta}$

$\sqrt{\alpha^2+\beta^2}\left[\frac{\beta^3}{\sqrt{\alpha^2+\beta^2}-\alpha} + \frac{\alpha^3}{\sqrt{\alpha^2+\beta^2}+\beta}\right]$

$$\sqrt{\alpha^2+\beta^2}\left[\frac{\beta^3(\sqrt{\alpha^2+\beta^2}+\alpha)}{\alpha^2+\beta^2+\alpha^2}+\frac{\alpha^3(\sqrt{\alpha^2+\beta^2}-\beta)}{\alpha^2+\beta^2-\beta^2}\right]$$

$$\sqrt{\alpha^2+\beta^2}\left[\frac{\beta^3(\sqrt{\alpha^2+\beta^2}+\alpha)}{\alpha^2+\beta^2-\alpha^2}+\frac{\alpha^3(\sqrt{\alpha^2+\beta^2}-\beta)}{\alpha^2+\beta^2-\beta^2}\right]$$

$$\sqrt{\alpha^2+\beta^2}\ [\beta\sqrt{\alpha^2+\beta^2}+\alpha\beta+\alpha\sqrt{\alpha^2+\beta^2}-\alpha\beta]$$

$= (\alpha+\beta)(\alpha^2+\beta^2)$

Sol.6 $\tan^{-1}+\cos^{-1}\dfrac{y}{\sqrt{1+y^2}} = \sin^{-1}\dfrac{3}{\sqrt{10}}$

$\tan^{-1}x+\tan^{-1}\left(\dfrac{1}{y}\right) = \sin^{-1}\dfrac{3}{\sqrt{10}} = \tan^{-1}3$

$\tan^{-1}\left(\dfrac{x+\frac{1}{y}}{1-\frac{x}{y}}\right) = \tan^{-1}3$

$\dfrac{xy+1}{y-x} = 3$

$xy + 1 = 3y - 3x$

$y = \dfrac{3x+1}{x-3}$

$y = \dfrac{3x+1}{3-x}$

Becoz it using poositive integral solution

x = 1, 2 only when possible

$x = 1 \quad y = \dfrac{3+1}{3-1}$

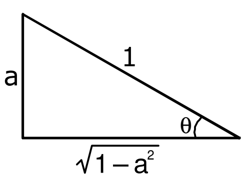
$x = 2 \quad y = \dfrac{7}{1} = 7$

(1, 2) (2, 7)

Sol.7 $x = \text{cosec}\tan^{-1}\cos\cot^{-1}\sec\sin^{-1}a$

Let $\sin^{-1}a = \theta$

$\sin\theta = a$



$\cos\theta = \sqrt{1-a^2}$

$\sec\theta = \dfrac{1}{\sqrt{1-a^2}}$

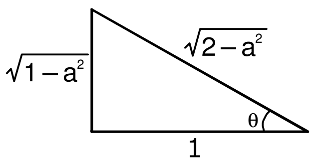
$= \text{cosec}\tan^{-1}\cos\cot^{-1}\sec\theta$

$= \text{cosec}\tan^{-1}\cos\cot^{-1} = \dfrac{1}{\sqrt{1-a^2}}$

$\cot^{-1}\dfrac{1}{\sqrt{1-a^2}} = \alpha$

$\cot\alpha = \dfrac{1}{\sqrt{1-a^2}}$

$\tan\alpha = \sqrt{1-a^2}$



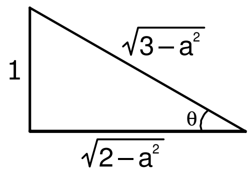
$\cos\alpha = \dfrac{1}{\sqrt{2-a^2}}$

$= \text{cosec}\tan^{-1}\cos\alpha$

$= \text{cosec}\tan^{-1}\dfrac{1}{\sqrt{2-a^2}}$

$\tan^{-1}\dfrac{1}{\sqrt{2-a^2}} = \beta$

$\tan\beta = \dfrac{1}{\sqrt{2-a^2}} = \text{cosec}\,\beta$



$x = \sqrt{3-a^2}$

$y = \sqrt{3-a^2}$

$x = 4$

Sol.9 We know

$\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$

so given equation can be written as

$(\sin^{-1}x)^3 + \left(\dfrac{\pi}{2}-\sin^{-1}x\right)^3 = \alpha\pi^3$

...(1)

at $\sin^{-1}x = t; t \in (-1, 1]$.

then $12t^2 - 6\pi t + \pi^2(1-8\alpha) = 0$

for no roots $D < 0 \quad \Rightarrow \alpha < \dfrac{1}{32}$

Sol.10 (a) $(\cot^{-1}x)^2 - 5\cot^{-1}x + 6 = 0$

Let $\cot^{-1}x = t$

$t^2 - 5x + 6 = 0$

$(t-2)(t-3) = 0$

$t < 3 \qquad t < 2$

$\cot^{-1} x > 3 \qquad \cot^{-1} x < 2$

$x < \cot 3 \qquad x > \cot 2$

$x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$

(b) $\sin^{-1} x > \cos^{-1} x$

$\sin^{-1} > \frac{\pi}{2}$

$\sin^{-1} > \frac{\pi}{4} \quad \Rightarrow \quad x \in \left(\frac{1}{\sqrt{2}}, 1\right]$

(c) $\tan^2 (\sin^{-1} x) > 1$

$x \in [-1, 1]$; $f(x)$ $|\tan x| > 0$

$\Rightarrow \quad x \in \left[-1, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right]$

Sol.11 $4(\tan^{-1} x)^2 - 8 \tan^{-1} x + 3 < 0$

$4t^2 - 8t + 3 < 0$

$4t^2 - 6t - 2r + 3 < 0$

$2t(2t-3) - 1(2t-3) < 0$

$(2t-3)(2t-1) < 0$

$$\frac{1}{2} < t < \frac{3}{2}$$

$$\frac{1}{2} < \tan^{-1} x < \frac{3}{2}$$

$$\tan\frac{1}{2} < x < \tan\frac{3}{2}$$

$4 \cot^{-1} x - \cot^{-1} x^2 - 3 \geq 0$

$a^2 - 4a + 3 \leq 0$

$a^2 - 3a - a + 3 \geq 0$

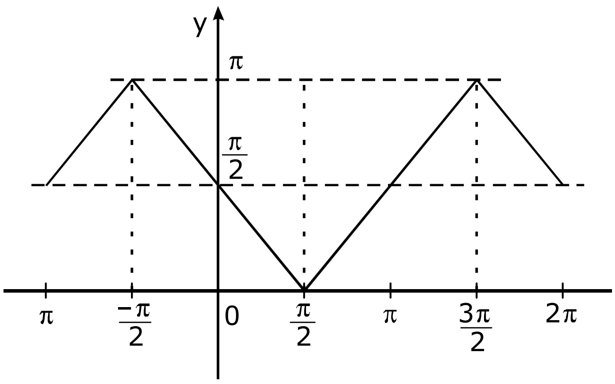
$(a-3)(a-1) \leq 0$

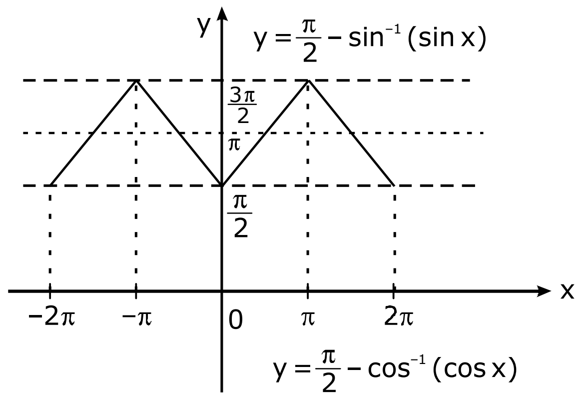
$1 \leq a \leq 3$

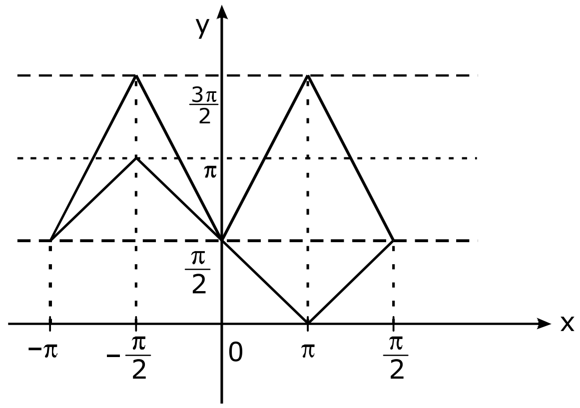
$1 \leq \cot^{-1} x \leq 3$

$\cot 1 \geq x \geq \cot 3$

Sol.12







So between $[-7\pi, 7\pi]$; $49A = 3388$.

Sol.13 $$\sum_{n=1}^{10} \sum_{n=1}^{10} \tan^{-1}\left(\frac{m}{b}\right) = kx$$

$$S = \sum_{n=1}^{10}\left(\tan^{-1}\frac{1}{n} + \tan^{-1}\frac{2}{n} + \tan^{-1}\frac{3}{n} + \ldots\ldots + \tan^{-1}\frac{10}{n}\right)$$

$$\sum_{n=1}^{10} \tan^{-1}\frac{1}{n} = \tan^{-1} 1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \ldots\ldots + \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{1}{10}$$

$$\sum_{n=1}^{10} \tan^{-1}\frac{2}{n} = \tan^{-1}\frac{2}{1} + \tan^{-1}\frac{2}{2} + \tan^{-1}\frac{2}{3} + \ldots\ldots + \tan^{-1}\frac{2}{10}$$

$$\sum_{n=1}^{10} \tan^{-1}\frac{10}{n} = \tan^{-1}\frac{10}{1} + \tan^{-1}\frac{10}{2} + \tan^{-1}\frac{10}{3} + \ldots\ldots + \tan^{-1}\frac{10}{10}$$

$$S = \left(10 \cdot \frac{\pi}{4}\right) + \left(\tan^{-1}\frac{1}{2} + \tan^{-1} 2\right)$$

$$+\left(\tan^{-1}\frac{1}{3}+\tan^{-1}3\right)+\left(\tan^{-1}\frac{1}{4}+\tan^{-1}4\right)$$

$$=\frac{5\pi}{2}+\frac{45\pi}{2} = 25\,\pi = k\pi$$

Sol.14 $\tan^{-1} r + \tan^{-1} S + \tan^{-1} t$

$$\tan^{-1}\left[\frac{r+s+t-rst}{1-rs-st-tr}\right]$$

$x(x - 2)(3x - 7) = 2$
$(x^2 - 2x)(3x - 7) = 2$
$3x^2 - 7x^2 - 6x^2 + 14x - 2 = 0$
$3x^3 - 13x^2 + 14x - 2 = 0$

$r + s + t = \frac{13}{3}$ $\quad$ $rst = \frac{2}{3}$

$\Sigma rs = \frac{14}{3}$

$$\tan^{-1}\left[\frac{\frac{13}{3}-\frac{2}{3}}{1-\frac{14}{3}}\right] = \tan^{-1}\left[\frac{11}{-11}\right]$$

$$\tan^{-1}[-1] = \frac{3\pi}{4}$$

Sol.15 $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right) < \pi - 3$

$$\sin\left(\frac{2x^2+4}{1+x^2}\right) > 3$$

$2x^2 + 4 > 3 + 3x^2$
$x^2 - 1 < 0$
$-1 < x < 1$

Sol.16 $2t = a + \frac{a^2}{t}$

$2t^2 = at + a^2$
$2t^2 - at - a^2 = 0$

$$t = \frac{a \pm \sqrt{a^2+8a^2}}{4}$$

$$t = \frac{a \pm 3a}{4}$$

$$t = \frac{a \pm 3a}{4}$$

(1) $t = \frac{a+3a}{4}$

$t = a$
$\cos^{-1} x = a$
$a = [0, \pi]$

(2) $t = \frac{a-3a}{4}$

$$t = \frac{-2a}{4}$$

$$t = \frac{-a}{2}$$

$$\cos^{-1} = \frac{-a}{2}$$

$$0 \le \frac{-a}{2} \le x$$

$$-\pi \le \frac{a}{2} \le 0$$

$$-2\pi \le a \le 0$$

Sol.17 $\sin^{-1} x < \frac{3\pi}{4}$

$$x > \sin\frac{3\pi}{4}$$

$$1 \ge x > \frac{1}{\sqrt{2}}$$

$$x \in \left(\frac{1}{\sqrt{2}}, 1\right]$$

Sol.18 $\sin^{-1}(-1) + \operatorname{cosec}^{-1}(-1)$

$$=\frac{3\pi}{2}+\frac{3\pi}{2} = 3\pi$$

Sol.19 If $\quad -1 \le x \le 1$
then $\quad -1 \le -x \le 1$

So $\quad \frac{3\pi}{4} \le \tan^{-1}(-x) \le \frac{5\pi}{4}$

Sol.20 by checking options

(D) $\sin^{-1}\sqrt{1-x^2}$

put $x = \cos\theta \Rightarrow \theta \in [0, \pi]$

because $x \in (0, 1) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

$\sin^{-1}\sin\theta = \theta = \cos^{-1} x$

Sol.21 by checking options

(C) Put $x = \sin\theta \Rightarrow \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

but as $x \in (0, 1) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

$\cos^{-1}\cos\theta = \theta = \sin^{-1} x$

Sol.22 by checking options

(D) put $x = \cos\theta$ $\quad\theta \in [0, \pi]$

but $x \in (-1, 0) \Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right)$

$\pi + \tan^{-1} \frac{\sqrt{1-x^2}}{x}$

$\pi + \tan^{-1}\tan\theta$

$= \pi + (\theta - \pi) = \theta = \cos^{-1} x$

EXERCISE – V HINTS & SOLUTIONS

Sol.1 Given function is

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

Function is defined, if

(i) $x(x+1) > 0$ $\because$ Domain of square root function.

(ii) $x^2+x+1 \geq 0$ $\because$ Domain of square root function.

(iii) $\sqrt{x^2+x+1} \leq 1$ $\because$ Domain of $\sin^{-1}$ function.

From (ii) and (iii)

$0 \leq x^2 + x + 1 \leq 1 \cap x^2 + x \geq 0$

$\Rightarrow$ $0 \leq x^2 + x + 1 \leq 1 \cap x^2 + x + 1 \geq 1$

$\Rightarrow$ $x^2 + x + 1 = 1 \Rightarrow x^2 + x = 0$

$\Rightarrow$ $x(x + 1) = 0 \Rightarrow x = 0, x = -1$

Sol.2 $\sin^{-1}\frac{x}{a} = \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

$\Rightarrow$ $\sin^{-1}\frac{142}{65\sqrt{5}} = \tan^{-1}\frac{142}{31}$

$2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\frac{5}{12}$

$3\tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\frac{11}{2}$

$S = 3\tan^{-1}\left(\frac{1}{2}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) + \sin^{-1}\frac{142}{65\sqrt{5}}$

$= \tan^{-1}\frac{11}{2} + \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{142}{31}$

$= \pi + \tan^{-1}\left(\frac{\frac{11}{2}+\frac{5}{12}}{1-\frac{55}{24}}\right) + \tan^{-1}\frac{142}{31}$

$= \pi - \tan^{-1}\frac{142}{31} + \tan^{-1}\frac{142}{31} = \pi$

Sol.3 $\sin^{-1}\frac{ax}{c} + \sin^{-1}\frac{bx}{c} = \sin^{-1}x$

$\sin^{-1}\left[\frac{ax}{c}\sqrt{1-\frac{b^2x^2}{c^2}} + \frac{bx}{c}\sqrt{1-\frac{a^2x^2}{c^2}}\right] = \sin^{-1}x$

$\therefore$ $x\left[a\sqrt{c^2-b^2x^2} + b\sqrt{c^2-a^2x^2} - c^2\right] = 0$

$x = 0$ is one solution

& $a\sqrt{c^2-b^2x^2} + b\sqrt{c^2-a^2x^2} - c^2 = 0$

$a^2\sqrt{c^2-b^2x^2} + b\sqrt{c^2-a^2x^2} = c^2$

$a^2(c^2 - b^2x^2) + b^2(c^2 - a^2x^2) + 2ab\sqrt{c^2-b^2x^2}$

$\sqrt{c^2-a^2x^2} = c^4$

put $a^2 + b^2 = c^2$

$\Rightarrow$ $abx^2 = \sqrt{c^2-b^2x^2}\ \sqrt{c^2-a^2x^2}$

$a^2b^2x^4 = c^4 - c^2(a^2 + b^2)x^2 + a^2b^2x^4$

$0 = c^4(1 - x^2) \Rightarrow x = \pm 1$ $x = \{0, -1, 1\}$

Sol.4 $\cos^{-1} 3\sqrt{3}\ x^2 = \frac{\pi}{2} - \cos^{-1}(\sqrt{6}\ x) = \sin^{-1}(\sqrt{6}\ x)$

$\cos^{-1} 3\sqrt{3}\ x^2 = \cos^{-1}(\sqrt{1-6x^2})$

$27x^4 = 1 - 6x^2$

$27x^4 + 6x^2 - 1 = 0$

$x^2 = -\frac{1}{3}$ & $x^2 = \frac{1}{9}$

(Reject) $x = \pm\frac{1}{3}$

$x = -\frac{1}{3}$ will not satisfy so $x = \frac{1}{3}$

Sol.5 We know that, $\sin^{-1}(\alpha) + \cos^{-1}(\alpha) = \frac{\pi}{2}$

Therefore, α should be equal in both functions.

$\therefore$ $x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots = x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots$

$\Rightarrow$ $\frac{x}{1+\frac{x}{2}} = \frac{x^2}{1+\frac{x^2}{2}}$ $\Rightarrow$ $\frac{x}{\frac{2+x}{2}} = \frac{x^2}{\frac{2+x^2}{2}}$

$\Rightarrow$ $\frac{2x}{2+x} = \frac{2x^2}{2+x^2} \Rightarrow 2x(2 + x^2) = 2x^2(2 + x)$

$\Rightarrow$ $4x + 2x^3 = 4x^2 + 2x^3$

$\Rightarrow$ $x(4 + 2x^2 - 4x - 2x^2) = 0$

$\Rightarrow$ Either $x = 0$ or $4-4x = 0$ $\Rightarrow x = 0$ or $x = 1$

$\because$ $0 < |x| < \sqrt{2}$, $\therefore$ $x = 1$ and $x \neq 0$

Sol.6 LHS = $\cos \tan^{-1} [\sin (\cot^{-1} x)]$

$$= \cos \tan^{-1} \left[\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$$= \cos\left(\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sqrt{\frac{x^2+1}{x^2+2}} = \text{RHS}$$

Sol.7 $f(x) = \sqrt{\sin^{-1}(2x)+\frac{x}{6}}$

$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$

$\frac{\pi}{2} \geq \sin^{-1} 2x \geq -\frac{\pi}{6}$

$-\frac{1}{2} \leq 2x \leq 1$

$-\frac{1}{4} \leq x \leq \frac{1}{2}$

$x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

Sol.8 Given, $\sin [\cot^{-1} (1 + x)] = \cos (\tan^{-1} x)$ ....(i)

and we know, $\cot^{-1} \theta = \sin^{-1}\left(\frac{1}{\sqrt{1+\theta^2}}\right)$,

and $\tan^{-1} \theta = \cos^{-1}\left(\frac{1}{\sqrt{1+\theta^2}}\right)$

$\therefore$ From Equation (i),

$$\sin\left(\sin^{-1}\frac{1}{\sqrt{1+(1+x)^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}}$$

$\Rightarrow$ $1 + x^2 + 2x + 1 = x^2 + 1$

$\Rightarrow$ $x = -\frac{1}{2}$

Sol.9 (A) If $a = 1, b = 0$, then $\sin^{-1} x + \cos^{-1} y = 0$

$\Rightarrow$ $\sin^{-1} x = -\cos^{-1} y$ $\Rightarrow$ $x^2 + y^2 = 1$.

(B) If $a = 1$ and $b = 1$, then

$\sin^{-1} x + \cos^{-1} y + \cos^{-1} xy = \frac{\pi}{2}$

$\Rightarrow$ $\cos^{-1} x - \cos^{-1} y = \cos^{-1} xy$

$\Rightarrow$ $xy + \sqrt{1-x^2}\sqrt{1-y^2} = xy$

$\Rightarrow$ $(x^2 - 1)(y^2 - 1) = 0$

(C) If $a = 1, b = 2$, then

$\sin^{-1} x + \cos^{-1} y + \cos^{-1} (2xy) = \frac{\pi}{2}$

$\Rightarrow$ $\cos^{-1} x - \cos^{-1} y = \cos^{-1} (2xy)$

$\Rightarrow$ $xy + \sqrt{1-x^2}\sqrt{1-y^2} = 2xy$

$\Rightarrow$ $x^2 + y^2 = 1$

(D) If $a = 2$ and $b = 2$ then

$\sin^{-1} (2x) + \cos^{-1} (y) + \cos^{-1} (2xy) = \frac{\pi}{2}$

$\Rightarrow$ $\cos^{-1} (2x) - \cos^{-1} (y) = \cos^{-1} (2xy)$

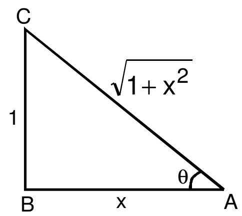
$\Rightarrow$ $2xy + \sqrt{1-4x^2}\sqrt{1-y^2} = 2xy$

$\Rightarrow$ $(4x^2 - 1)(y^2 - 1) = 0$

Sol.10 We have, $0 < x < 1$

Let $\cot^{-1} x = \theta$

$\Rightarrow$ $\cot \theta = x$



$\Rightarrow$ $\sin \theta = \frac{1}{\sqrt{1+x^2}}$

$= \sin (\cot^{-1} x)$

and $\cos \theta = \frac{x}{\sqrt{1+x^2}} = \cos (\cot^{-1} x)$

Now, $\sqrt{1+x^2}\, [\{x \cos (\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 - 1]^{1/2}$

$$\sqrt{1+x^2}\left[\left(x.\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\left[\left(\frac{1+x^2}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{1+x^2}\,[1 + x^2 - 1]^{1/2} = x\sqrt{1+x^2}$$

Answer Ex–I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. D	**2.** D	**3.** C	**4.** D	**5.** B	**6.** D	**7.** A
8. C	**9.** B	**10.** D	**11.** C	**12.** A	**13.** D	**14.** B
15. B	**16.** C	**17.** A	**18.** D	**19.** B	**20.** B	**21.** A
22. B	**23.** C	**24.** C	**25.** A	**26.** A	**27.** B	**28.** C
29. D	**30.** B	**31.** B	**32.** A	**33.** C	**34.** B	**35.** B

Answer Ex–II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. A,B	**2.** C,D	**3.** B,D	**4.** A,B,C	**5.** B,C,D	**6.** A,C	**7.** A,C
8. A,C,D	**9.** A,B,C	**10.** B,C	**11.** B,C	**12.** A,C	**13.** A,D	

Answer Ex–III SUBJECTIVE QUESTIONS

1. 5 **2.** **(i)** $-\sin 1 < x \le 1$, **(ii)** $\cos 2 < x \le 1$, **(iii)** No solution

3. **(i)** $-\frac{\pi}{6}$, **(ii)** $-\frac{\pi}{3}$, **(iii)** $\frac{3\pi}{4}$, **(iv)** $\frac{\pi}{4}$ **4.** **(i)** $5 - 2\pi$, **(ii)** $4\pi - 10$, **(iii)** $2\pi - 6$, **(iv)** $4\pi - 10$, **(v)** $\frac{17\pi}{20}$

5. $\sin^{-1}(\sin\theta) = \begin{cases} \theta - 2\pi, & \frac{3\pi}{2} \le \theta \le \frac{5\pi}{2} \\ 3\pi - \theta, & \frac{5\pi}{2} < \theta \le 3\pi \end{cases}$; $\cos^{-1}(\cos\theta) = \begin{cases} 2\pi - \theta, & \frac{3\pi}{2} \le \theta < 2\pi \\ \theta - 2\pi, & 2\pi \le \theta \le 3\pi \end{cases}$;

$\tan^{-1}(\tan\theta) = \begin{cases} \theta - 2\pi, & \frac{3\pi}{2} < \theta < \frac{5\pi}{2} \\ \theta - 3\pi, & \frac{5\pi}{2} < \theta \le 3\pi \end{cases}$; $\cot^{-1}(\cot\theta) = \begin{cases} \theta - \pi, & \frac{3\pi}{2} \le \theta < 2\pi \\ \theta - 2\pi, & 2\pi < \theta < 3\pi \end{cases}$

6. **(i)** $x = \frac{1}{\sqrt{3}}$, **(ii)** $x = 2$ **7.** $\frac{1+xy}{x-y}$ **9.** 2 **10.** $-\pi$ **11.** $x = 4y^2$

13. **(i)** $\frac{1}{\sqrt{3}}$, **(ii)** 1, **(iii)** $\frac{5\pi}{6}$, **(iv)** $-\frac{\pi}{3}$, **(v)** $\frac{4}{5}$, **(vi)** $\frac{17}{6}$ **14.** **(i)** $\frac{1}{2}$, **(ii)** -1, **(iii)** $-\frac{\pi}{4}$, **(iv)** $\frac{2\pi}{3}$, **(v)** $\frac{4}{5}$, **(vi)** α

15. **(iv)** $(-\infty, \sec 2) \cup [1, \infty)$

16. **(i)** $-1/3 \le x \le 1$, **(ii)** $\{1, -1\}$, **(iii)** $1 \le x \le 4$, **(iv)** $x \in (-1/2, 1/2)$, $x \ne 0$, **(v)** $(3/2, 2]$, **(vi)** $\{7/3, 25/9\}$

(vii) $(-2, 2) - \{-1, 0, 1\}$, **(viii)** $\{x \mid x = 2n\pi + \frac{\pi}{6}, n \in I\}$ **17.** $\left[\frac{\sqrt{3}}{2}, 1\right]$

19. **(i)** $x = \frac{1}{2}\sqrt{\frac{3}{7}}$, **(ii)** $x = 3$, **(iii)** $x = 0,\ \frac{1}{2}, -\frac{1}{2}$, **(iv)** $x = \frac{3}{\sqrt{10}}$, **(v)** $x = 2 - \sqrt{3}$ or $\sqrt{3}$, **(vi)** $x = \frac{1}{2}$, $y = 1$,

(vii) $x = \frac{a-b}{1+ab}$ **20.** 53 **23.** $6\cos^{-1}x - \frac{9\pi}{2}$, so $a = 6$, $b = -\frac{9}{2}$

24. **(i)** $\frac{\pi}{2}$, **(ii)** $\frac{\pi}{4}$, **(iii)** arc cot $\left[\frac{2n+5}{n}\right]$, **(iv)** arc tan $(x + n)$ – arc tan x, **(v)** $\frac{\pi}{4}$

25. **(i)** $x = n^2 - n + 1$ or $x = n$, **(ii)** $x = ab$, **(iii)** $x = \frac{4}{3}$

26. **(i)** $(\cot 2, \infty) \cup (-\infty, \cot 3)$, **(ii)** $\left(\frac{\sqrt{2}}{2}, 1\right]$, **(iii)** $\left(\frac{\sqrt{2}}{2}, 1\right) \cup \left(-1, -\frac{\sqrt{2}}{2}\right)$

Answer Ex–IV ADVANCED SUBJECTIVE QUESTIONS

3. $-\pi$ **4.** $6\cos^{-1}x - \frac{9\pi}{2}$, so $a = 6$, $b = -\frac{9}{2}$

5. **(a)** arc cot $\left[\frac{2n+5}{n}\right]$, **(b)** $\frac{\pi}{4}$, **(c)** arc tan $(x + n)$ – arc tan x, **(d)** $\frac{\pi}{4}$, **(e)** $\frac{\pi}{2}$

6. $(\alpha^2+\beta^2)(\alpha+\beta)$ **7.** $K = 2$, $\cos\frac{\pi^2}{4}, 1$ & $\cos\frac{\pi^2}{4}, -1$ **8.** $x = 1$, $y = 2$, & $x = 2$; $y = 7$

9. $X = Y = \sqrt{3-a^2}$ **10.** (A)→P, Q, R, S ; (B)→P, Q ; (C)→P, R, S ; (D)→P, R, S

12. **(a)** $(\cot 2, \infty) \cup (-\infty, \cot 3)$, **(b)** $\left(\frac{\sqrt{2}}{2}, 1\right]$, **(c)** $\left(\frac{\sqrt{2}}{2}, 1\right) \cup \left(-1, -\frac{\sqrt{2}}{2}\right)$ **13.** $\left(\tan\frac{1}{2}, \cot 1\right]$

14. 3388 **15.** $k = 25$ **16.** $\frac{3\pi}{4}$ **17.** $x \in (-1, 1)$ **18.** $a \in [-2\pi, \pi] - \{0\}$

19. A **20.** C **21.** B **22.** D **23.** C **24.** D

Answer Ex–V JEE PROBLEMS

1. C **2.** π **3.** $x \in \{-1, 0, 1\}$ **4.** $x = 1/3$ **5.** B **7.** D

8. A **9.** (A)→P ; (B)→Q ; (C)→P ; (D)→S **10.** C